APPENDIX C
THE COVENANT TRUST MODEL
SELF-CERTIFICATION OF DIGITAL SIGNATURE KEYS BY CONTRACT

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A. THE COVENANT TRUST MODEL

I. BACKGROUND

With the Electronic Signatures in Global and National Commerce Act of 2000, the U.S. Congress gave digital signatures the same legal validity as an ink signature on a piece of paper. Now, the sender of an email message, word processing document, or any other type of electronic record that can be construed as a written contract can be legally bound to that record if the recipient can prove that the sender authenticated the record.

Electronic records that are signed with digital signatures can be proven, to a very high level of certainty, to be authenticated by the person who caused the digital signature to be applied to the record. The digital signature can only be applied with a private key, which is an incredibly large number that uniquely corresponds to another incredibly large number, called a public key. The private key, as its name implies, is kept a strict secret by the person who uses it to sign his or her digital signature. Strong cryptographic software ensures that it is "computationally infeasible" (i.e., very difficult, even with very fast computers) to derive the private key from the public key. When a person signs an electronic record with their private key, a digital signature code is produced that anyone can verify against the public key, which is publicly accessible. The slightest change in a document so signed will cause the digital signature to no longer match the document.

The cryptography used in digital signatures is very strong and nearly impossible to tamper with, at least with current technology. But a very old problem remains that technology alone cannot entirely solve. That problem is trust.

The trust problem in digital signatures can be summarized as follows: How do you know that the public key really belongs to the person who says it belongs to him or her? Anyone can create a public key and call it someone else's, then use the corresponding private key to create forged electronic records. The 1998 edition of The Global Trust Register, a printed directory of public keys published by a group of cryptography experts, states the problem as follows: "[T]here is no cheap and effective

way for Internet users to check the validity of public keys on which they may wish to rely."

The experts who wrote The Global Trust Register made that statement in spite of the many efforts by Certification Authorities (CAs) to deploy a "hierarchical trust" model, where trusted third parties check out the identity of persons who own private/public key pairs. A CA such as Verisign, Entrust, or Thawte will add its digital signature to a public key if the public key is tied to the name of a person who physically appears with proper documentation to prove their identity. Recipients of documents signed with the certified public key are then expected to trust that the CA has done its job and that the public key really came from the person whose name is tied to it.

But what happens when one of the many employees at the CA doesn't do his or her job properly? Who is liable for the recipient's reliance on a forged document promising delivery of 10,000 widgets for $1M when the sender has pocketed the money and run, completely anonymously due to the faceless nature of the Internet? The recipient cannot sue the sender if the recipient doesn't know the sender really was. The recipient's only course of action is to sue the CA for not doing its job. CAs try to avoid liability with disclaimer language in their Certification Practice Statements.

What about tort claims against the CA? Here's what the text Certification Authority Liability Analysis has to say about that:

> A CA's liability for tort claims based on negligence **may be limited** by the so-called "economic loss doctrine." The economic loss doctrine provides that claims for purely economic losses based on product defects are not recoverable in tort. The rule holds simply that tort liability does not arise for pure economic loss, but only for personal injury or property damage. The principles behind this rule are that protecting personal injury and property damage claims are more important social policies than pure economic (business) losses, and that economic losses are better protected by negotiated contract allocations rather than through generalized tort law. (Certification Authority Liability Analysis Section 1.1, American Banker's Association, 1998, emphasis added.)

In addition to the problems with "hierarchical trust" that should now be apparent, reliance on the Certification Authority as a trusted third party requires the CA to have an established reputation and to keep its digital house in order for a long time. It doesn't do much good to have a "trusted" third party certifying a digital signature if that third party disappears, loses data, or is found out to have some serious security breach in its infrastructure.

In view of these problems, a system is needed that will translate the direct trust from signer to recipient that self-authenticating ink signatures now provide into the realm of digital signatures. The solution, it turns out, is combining technology with the

trusted authentication that ink signatures and signature witnesses have established over hundreds of years of history.

II. THE COVENANT - AN ANCIENT CONCEPT APPLIED TO TECHNOLOGY

The Covenant Trust Model relies on a person's self-certification of his or her public key and a covenant by that person not to repudiate the public key. The "Covenant of Non-repudiation" legally binds the owner of the public key to any digital signatures created with the corresponding private key. Thus, the liability for proper usage of the private key is placed on the shoulders of the person owning the public key, where it belongs, and legal reliance can be placed upon the public key and any electronic record signed with the corresponding private key.

The covenant is made in an Authentication and Certification Instrument (ACI), a legally signed paper document that contains an identification code positively identifying the public key in question. The document is signed in ink and witnessed by a notary public, thus invoking an authentication system whose trust has been established and is universally recognized by our legal system. An example ACI (see Appendix A-1) contains the following text:

> I acknowledge and understand that the consequence of executing this authorization and certification instrument ("Authorization") is that any electronic record accompanied by a digital signature that uniquely corresponds to both the document and the Public Key was signed by me, with a negligible level of doubt. I covenant with any bearer of this Authorization or facsimile copy thereof not to repudiate such digital signature unless I communicate (directly or indirectly) a revocation of the Public Key to the bearer in writing before the signature date.

The ACI includes security features, discussed below, that make it extremely difficult to forge with identification of a different public key, even in a facsimile copy. A person receiving a copy of the ACI (from the signer, from the Internet, wherever) is in possession of a legal instrument that authenticates a public key without the need for trusted third parties. The role of a third party, if one is used at all, is simply to distribute facsimile copies of the ACI. For additional security, the third party can apply its digital signature to the copies of the ACI it distributes to certify them as true copies of the original signed in ink. For example, the third party can authenticate PDF or TIFF files containing facsimile copies of ACIs with a standard SSL (Secure Sockets Layer) certificate issued by a conventional CA.

The conventional "hierarchical trust" model attempts to establish a chain of authenticity to supposedly trusted third parties who are presumed to be doing their jobs properly. In contrast, the covenant trust model establishes a chain of authenticity to a legal covenant, signed with a notarized ink signature on an ACI, in which a public key

owner promises not to repudiate digital signatures corresponding to that public key. The chain of authenticity can begin with initial reliance on the security features of a facsimile copy of the ACI and distribution of the ACI via a trusted web site, email sender, or remote-access viewing software. Higher up on the chain of authenticity, and still convenient to obtain, is digitally-signed certification of the copy by a trusted certifier. Still higher on the authenticity chain is the availability of ink-signed certified copies of the ACI by the original signer or, for a fee, by a trusted certifier. The ultimate link in the chain of authenticity can be provided by making the original notarized, ink-signed ACI paper available for inspection by experts, judges, juries, or attorneys during dispute resolution.

B. IMPLEMENTATION OF COVENANT TRUST VIA THE INTERNET

I. OVERVIEW

A new type of "Certification Authority" will be deployed at **SelfCertify.com** based on the covenant trust model. Selfcertify.com (discussed here in the present tense for convenience) is a certification authority only in the sense that it registers public keys and the identity of persons who claim to own those keys, and certifies that copies of ACIs it distributes are true copies of originals in its possession. It does not certify the identities of the person claiming to own the public keys - those persons make that certification themselves in the ACI.

In addition to registering public keys and distributing ACIs for authentication of those keys, SelfCertify.com can provide standardized digital certificates (e.g., using the X.509 standard) to ensure that its subscriber's public keys can be validated in a manner compatible with conventional public key infrastructure. Again, SelfCertify.com does not pretend that the trust imparted by its digital certificates is based on its confirmation of the identity of its subscribers. Instead, SelfCertify.com makes a policy of only issuing certificates for public keys that subscribers have self-certified with their notarized ink signatures in ACI documents. By signing a public key with its X.509 certificate, SelfCertify.com simply indicates that it has reviewed the original ink ACI and that a copy of the document can be freely downloaded from its Web server.

The use of X.509 or other standard certificates permits SelfCertify.com to live in the world of conventional CAs even though it is based on an entirely different trust model. Users who accept the covenant trust model can install SelfCertify.com's root CA certificate (the "grandfather" certificate that validates all of its individual certificates) into their Web browsers and e-mail applications. As the covenant trust model gains

acceptance in E-commerce, the manufacturers of Netscape Navigator and Internet Explorer can be expected to incorporate SelfCertify.com's root CA certificate into their browsers, alongside the certificates of VeriSign, entrust, and dozens of other CAs. Subscribers who use PGP (Pretty Good Privacy) and are looking for a way to validate their public keys outside PGP's "web of trust" model can submit their public keys to SelfCertify.com for it to be signed by SelfCertify.com's own PGP signature.

Because covenant trust does not require a trusted third party, subscribers' public keys can be validated directly from the subscriber's ACI. The public key of a SelfCertify.com subscriber can be validated by freely downloading a copy of the subscriber's ACI and checking its positive identification of the public key. Thus, no CA certificate is required at all. In fact, subscribers can directly distribute copies of their ACI to anyone who will be relying on signatures corresponding to their public keys.

II. EXAMPLE TRANSACTION USING SELFCERTIFY.COM

Below is a brief description of an example transaction based on covenant trust. In this example transaction, SelfCertify.com serves as a third party for the following:

1. Freely distributing a compact cryptographic software module to signer and recipient with instructions for secure use. The parties use the software for generation of the signer's private/public key pair, generation of the signer's digital signature on an electronic record, and validation of the digital signature against the signer's public key.
2. Accepting credit card payment (with SSL encryption), public key codes, and full legal names of new subscribers to SelfCertify.com.
3. Issuing blank ACIs to new subscribers, upon payment, with instructions for use.
4. Scanning original signed ACIs received from new subscribers and posting digitally certified copies on the web for free downloading.
5. Retaining original ACIs in a vault for inspection by experts, judges, juries, or attorneys during dispute resolution.

For convenience, this example refers to a widget vendor named Alice and a purchaser named Bob. (These names seem to be used in just about every published example of cryptographic transactions.) Alice wishes to sign a purchase agreement acknowledging Bob's payment of $1M for 10,000 widgets and promises to deliver the widgets immediately. Bob wants to make sure that Alice, the president of Widgets Inc., is the person signing the agreement and not some "man-in-the-middle" imposter.

- **Signer Enrollment**

Alice visits SelfCertify.com and quickly downloads a copy of "SelfCertify", a simple, compact, secure, and free cryptographic software application for Windows 98/NT/2000, with versions available for various other operating systems. The SelfCertify software installs to the Windows tray as an icon, with various functions selectable by right-clicking on the icon. If she wishes to avoid the need for installation, Alice has the option of simply downloading a single executable file to her desktop and running it from there. For maximum convenience (but possibly less security), a Java version of the software can be offered for execution in a web browser. Because SelfCertify.com serves its pages under SSL with a certificate issued by a conventional CA, Alice is assured that the software is authentic and trustworthy. For additional assurance, Alice reviews statements on the security of the software, written and digitally signed by various cryptographic experts, and validates the signatures of the statements before relying on the software.

Alice then follows the procedures outlined on SelfCertify.com for generating a public key from a secure passphrase. (See Appendix X.) She then gets out her credit card and subscribes to SelfCertify.com with her credit card number, public key code, and full legal name.

Selfcertify.com then issues Alice a custom-generated PDF file, from which Alice obtains two printed pages. The first page is a blank ACI with a space for her driver's license or other photographic ID and the second page is customized security paper with Alice's key code printed repeatedly in the background in an outline font.

Alice tapes her driver's license to the blank ACI in the space provided and places it on the glass of her photocopier, with the security paper at the top of her photocopier's paper supply. She then photocopies the blank ACI to produce an ACI, ready for her signature, with outline digits of her key code throughout its background.

Alice then checks the key code against her public key to make sure it is accurate, goes to the Notary Public down the hall, and executes the ACI in the presence of the notary. The notary examines Alice's driver's license, notes (in the ACI) any security features of it such as a hologram or colored background lines, and signs and stamps the ACI. Alice has now entered into a legally binding covenant with any person bearing the ACI or a facsimile copy of it. (So that she can keep a copy for her files and make certified copies herself, Alice elects to prepare and execute two original copies of the same ACI before the notary.)

Alice mails the executed ACI to SelfCertify.com. Within a few days, SelfCertify.com scans the ACI and posts a copy of it on its web site in PDF or TIFF format. Selfsertify.com stores the original ACI in a vault for possible inspection in the future by experts, judges, juries, or attorneys during dispute resolution. Selfcertify.com then emails Alice the following message:

> Your Authorization and Certification Instrument (ACI) has been recorded and you are now listed as a fully enrolled subscriber of **SelfCertify.com** with key ABC01. Once you enter the enrollment password "3f8u2b" in your SelfCertify software, your software will automatically download the latest copy of our public key registry (now including your key) and will automatically validate your digital signatures with the following text in any messages you sign: "The following text has been signed with a public key registered as key ABC01 at SelfCertify.com. Alice B. Costas has signed a written covenant not to repudiate digital signatures created with this public key. To view a copy of this document, click here. The code of this public key is BD7D F2FD EC1C DF14 4811 574F F7CE 7D1E 6EB6 F7E9 CCF7 208B." Persons relying on your digital signature will be able to easily download and inspect a copy of your ACI to legally bind you to that signature.

- **Signer's Digital Signature of Electronic Record**

In her email software, Alice selects the text of her purchase agreement with Bob and right-clicks on the SelfCertify icon in the Windows tray. She then selects the menu item "sign" and, when prompted, enters her private key passphrase. She will probably have to look the passphrase up from a piece of paper in her purse the first few times she uses it. Later, she will put the piece of paper in her safe or destroy it if she trusts her memory enough. If she forgets or loses the passphrase, it's not a big deal. She only needs to create another public key from a new passphrase, cancel her original ACI, and request another one to continue signing records.

As soon as Alice has entered her passphrase, the text she selected in her HTML-formatted email is replaced by text that is identical (including any formatting) except for a block of hexadecimal codes and the following statement in a reduced-size font:

> I, Alice B. Costas, have signed this document with my public key, which is registered as key ABC01 at **SelfCertify.com**. To verify this signature, click on http://SelfCertify.com/validate to download a compact, virus-free signature verification program that confirms the signature and public key. The software will allow you to obtain a copy of a paper document that you can use to legally bind me to this digital signature. You can also independently validate the public key by clicking on http://SelfCertify.com/?ABC001 to view a digitally certified copy of the document.

The formatting of the original text is preserved in the signed version. There is no header to the block of signed text because the SelfCertify software automatically calculates the beginning of the signed text block based on the number of signed characters, which is recorded in the signature block. Alice is free to select only a portion

of the text for signature. For example, she may choose not to include letterhead at the top of her letters in the block of text she signs.

Alice can also use S/MIME email software such as Netscape Messenger or Outlook Express to sign email messages using conventional, standardized digital signature technology and the Covenant Trust model, without the need for the SelfCertify.com software. However, she needs to sign an ACI with the SHA1 fingerprint of her S/MIME public key (called a "Digital ID") to authenticate it under the Covenant Trust model. SelfCertify.com then can issue a certificate for her S/MIME public key to authenticate it, based on her ACI.

- **Recipient's Validation of Digital Signature**

Bob receives Alice's digitally signed purchase agreement and downloads the SelfCertify software from the link provided in Alice's signature block. He also downloads a copy of her ACI. Once the software has been installed as an icon, Bob selects Alice's entire e-mail and right-clicks on the icon, then selects "Verify." A window pops up that says:

> The following text has been signed with a public key registered as key ABC01 at SelfCertify.com. Alice B. Costas has signed a written covenant not to repudiate digital signatures created with this public key. To view this paper, click **here**. The code of this public key is BD7D F2FD EC1C DF14 4811 574F F7CE 7D1E 6EB6 F7E9 CCF7 208B.

Since this is a $1M deal and he has never used the software before, Bob is not content with the software's assertion that Alice has entered into a legally binding covenant not to repudiate her digital signature with this key. Plus, Bob wants to have his lawyer look over the language of the covenant. So he clicks on the "here" link and a viewer window pops up with a TIFF copy of Alice's ACI. He prints out the ACI, notes that Alice's signature (which he recognizes from previous paper-based contracts) has been notarized and that the key code in the ACI is reproduced throughout the background of the document as vertically oriented digits in various outline fonts. The digits intermingle with the signatures, notary stamp, handwritten annotations, and images from Alice's driver's license. The key code digits even show up in the background of Alice's photograph in her driver's license.

Bob needs no further convincing that Alice was the one who signed purchase agreement. His lawyer, however, wants him to check out SelfCertify.com's SSL certificate for the copied ACI. Bob downloads the ACI copy from SelfCertify.com and, with the image of the ACI in his Web browser, clicks on the "security" button of the

browser. The browser provides a certificate issued to SelfCertify.com from a major CA, and Bob's lawyer is satisfied.

If Alice uses S/MIME to digitally signed her message, Bob can simply trust her S/MIME "Digital ID" based on the certificate SelfCertify has issued for it. Thus Alice and Bob can use the Direct Trust model with S/MIME signatures and conventional digital certificates, trusting selfcertify.com as a CA only for inspecting and verifying Alice's CA against the standard covenant language of the ACI, which is published at SelfCertify.com.

Alternatively, Bob can download and review Alice's ACI for her "Digital ID" from the web site of SelfCertify.com. If Bob chooses to download Alice's ACI, he will need to open Alice's "Digital ID," look for her SHA1 fingerprint, and compare it to the fingerprint printed on her ACI. This alternative procedure, while requiring an extra step, provides S/MIME signatures based more directly on the Covenant Trust model, moving closer to the ultimate link in the chain of authenticity, which is the original notarized, ink-signed ACI paper.

C. THE UNDERLYING TECHNOLOGY

The following is a brief listing of various aspects of the inventions discussed in this appendix:

- The key code in the ACI can be printed throughout the background of the entire paper as vertically oriented digits in various outline fonts. The font types, sizes, spacings, and line spacings are varied pseudorandomly in each ACI to make it difficult for an attacker to create an identical field of digits, which the attacker could use to remove the digits (by an XOR operation) from the ACI and substitute his or her own digits. Every bit of text and authenticating indicia in the ACI has background digits running through it. This feature (and possibly other features such as varying the spacing between digits of the text in a coded manner) protects both the signer of the ACI and the person relying on the ACI.
- The ACI can be created with a two-step procedure using a first page that is a blank ACI with a space for her driver's license or other photographic ID and a second page that is customized security paper with the subscriber's public key code printed repeatedly in the background in an outline font. The subscriber tapes his or her photo ID to the blank ACI in the space provided in places it on the glass of her photocopier, with the security paper at the top of the photocopier's paper supply. The blank ACI is then photocopied to produce an

ACI, ready for the subscriber's signature, with outline digits of her key codes throughout its background.

- The ACI can include language that makes it the only printed document of its type that can be accepted as valid. Additional ACIs can be signed electronically for additional keys, but they must be signed with the key that is certified in the original paper ACI. Selfcertify.com attaches digitally signed ACIs (for a fee) to the PDF or TIFF file in which it distributes the original paper ACI. By ensuring that the original printed document disclaims all other documents purporting to bear the singer's handwritten signature, a "strength in numbers" validity system is established that gives the authenticity of a widely distributed ACI, publicly available from a trusted server, far more weight than a single forged copy having a different key code. This feature helps to protect the signer of the ACI.
- The SelfCertify software can employ an ECDSA public key signature system with NIST Elliptic Curve P-192 (equivalent to 80-bit key length of symmetric cipher). The elliptic curve is described by a GF(p) field, where p is prime, to avoid recent attacks on elliptic curves from GF(2^m), where m is a composite of smaller primes. See Smart, N. et al., "Constructive and Destructive Facets of Weil Descent on Elliptic Curves," HP Technical Report HPL-2000-10, 17 January 2000.) A 192-bit public key can be represented by 12 groups of 4 hexadecimal digits. The short key length made possible by elliptic curve cryptography makes it easy for a recipient to visually verify the entire key code against the printed text of an ACI and the background security digits.
- The subscriber can be instructed to use a standardized, pronounceable passphrase made of "pseudowords" with alternating consonants and vowels. The passphrase is designed to be relatively easy to memorize, pronounce, and type and is very secure, with an entropy of about 2^{64}. The passphrase is created with simple, secure system using a piece of paper and a paper clip for random selection of digits.
- A SHA-1 hash of the passphrase can be used as the private key, with the subscriber's full legal name (from the SelfCertify.com directory) incorporated (transparently to the signer) into the passphrase as "salt." The use of salt prevents passphrase attacks using pre-computed hashes of passphrases within the standardized ~2^{64} passphrase space.
- Formatting of signed text can be preserved after signing. The added text of the signature block is formatted in an unobtrusive font that does not detract from the appearance of the signed text. The text in the signature block includes a data field

with the number of characters being signed, which avoids the need for a distracting header block (e.g., "-----BEGIN PGP SIGNED MESSAGE-----" in PGP). Documents can also be signed as files, in which case the signature resides in a separate ".SIG" file, as is conventional.

- ACIs can be automatically opened from the software's signature validation window, based on the identification information in the signature block, and displayed or printed from a compact viewing window.

# # #

Alice wishes to become a subscriber to SelfCertify.com so that Bob will rely on her public key. However, she doesn't wish to go through the hassle of having a paper document sent to her and having it signed in the presence of a notary. She also wants people to be able to authenticate her public key by hearing a simple recorded statement by her. So, she chooses the "Verbal ACI" option on the SelfCertify.com Web site and enters her phone number and the fingerprint of her public key into the form. The Web site then lists a phone number and an access code and invites her to call the number.

She dials the number (making sure that call blocking is disables so that SelfCertify.com can detect the phone number she's calling from) and enters the access code using the touchtone keys of her telephone. She then enters into a brief oral exchange with a computer or human operator at SelfCertify.com. The exchange goes something like this:

SC: This telephone call is being recorded for the permanent records of SelfCertify.com, for the purpose of authenticating a public key you are certifying with SelfCertify.com. If you consent to this recording and proceeding with the certification process, please state "I agree" and then recite your full legal name and mailing address.

Alice: I agree. My name is Alice P. Costas, and my address is 537 Main Street, Anytown Arizona 12345.

SC: Now that we have your agreement to record this telephone call and proceed, we will ask that you carefully read the terms of the "Authentication and Certification Instrument." You will be asked to agree to the terms of that document, and your recorded verbal agreement will legally bind you to those terms as if you had signed the document with your ink signature. Please state "Yes, it is" to confirm with the statement entitled "Authentication and Certification Instrument" is now displayed on your web browser at **https://www.selfcertify.com/aci32776** and that the document refers to a public key with fingerprint 2355 7782 1193 8001. You will be given an opportunity to read the document in a minute if you haven't already done so. Right now, we just ask you to confirm that the document is being displayed.

Alice: Yes, it is.

SC: Now we will ask you to ensure that you have read the document. We recommend that you print the document for your records, as you will be bound to its terms if you proceed. Please say "I have read the document" when you have done so.

Alice: Yes, I've read the document.

SC: Now please confirm your legally binding agreement with the terms of the document entitled "Authentication and Certification Instrument, " displayed on your web browser at **https://www.selfcertify.com/aci32776** and referring to a public key with fingerprint 2355 7782 1193 8001, on this___ day of _____,___ by stating "Yes, I agree to the terms of the document."

Alice: Yes, I agree.

SC: Sorry, you need to state exactly, "Yes, I agree to the terms of the document."

Alice: Yes, I agree to the terms of the document.

SC: Thank you. This includes your verbal certification of your public key. Thank you.

<End of Recording>

SIGNED MEDIA STREAMS

This invention is another aspect of the general concept of calculating a digital signature based on all of the contents of an electronic record except an excluded signature portion. (The general concept advantageously gets around the circular problem of a document essentially signing itself.)

With modern technology, it is difficult to place trust in the authenticity of a video or audio recording. Portions of the recording can be digitally modified in a way that even a careful observer cannot detect. One advantageous aspect of this invention permits a video or audio recording to be validated without requiring special a recording format. Another advantageous aspect of this invention performs frame-by frame authentication of a recording to ensure that the observer is alerted to unauthenticated portions of the recording.

An attached page includes two figures, one depicting the spectrum of an audio recording with an out-of-band transmission of digital signature information, and the other depicting the frame-by frame computation and transmission of digital signatures. During frame T2, a digital signature S1 is computed based on digital samples of the recording within frame T1, and the signature S1 is transmitted (along with other digital samples of the recording) in frame T3. During frame T3, a digital signature S2 is computed based on digital samples of the recording within frame T2, and the signature S2 is transmitted (along with other digital samples of the recording) in frame T4 (not shown). Optionally, digital signature S2 can include (or consist of) an aggregate signature formed from a bitwise modulo sum of the signature and the previous signature.

Advantageously, the video or audio recording can be transmitted and stored independent of any specific digital format, as long as the modulated digital signature information is faithfully conveyed along with the recording information. To allow for degradation of the recording, the signature is preferably computed based on truncated samples. An example of a truncated sample is an audio sample (e.g., noisy 16 bits) that is set to the nearest value within a truncated binary set (e.g., 8 bits). The likelihood of a noisy value being pushed over a boundary between value within the truncated set is small, about 2^(x-y) where x is the full set size in bits and y is the truncated set size in bits.

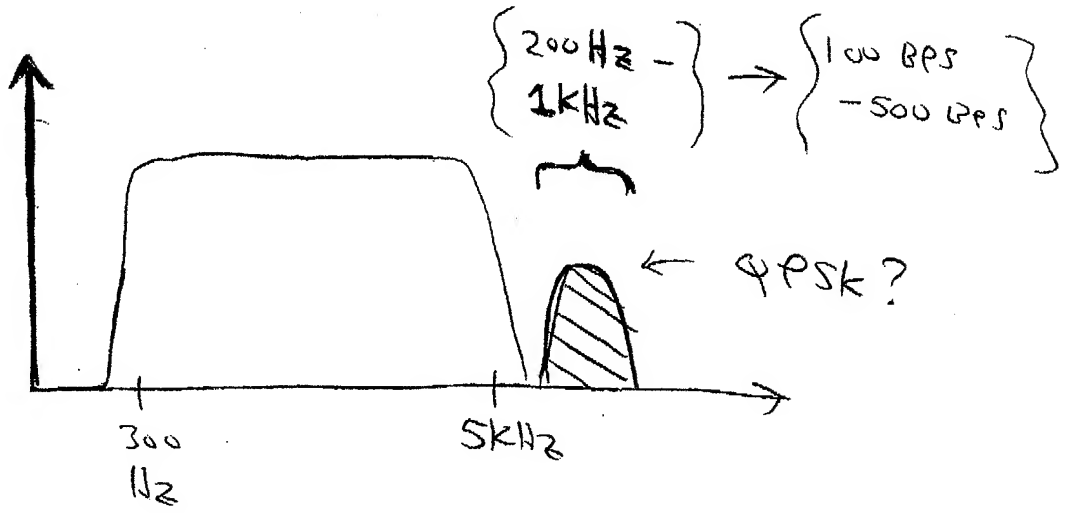
With x-y=8, the likelihood is 1 in 255, which represents a fairly significant probability of signature error in a given frame. Consequently, the number of samples in a frame should be kept fairly small.

More preferably, speech statistics (i.e., "feature vectors") used in speech recognition (13 spectral magnitude values within 10 ms frames) can be derived from the

recording. If the derivation of the statistics can be made robust in the presence of noise, less signature errors will result.

The media player can authenticate the media stream by including a visually intuitive authentication display. The display can take into account a running statistic of frame authentications, for example by slightly decreasing a "gas gauge" bar for each signature error in a moving average of 32 frames. If frames are short enough, the speech content will not be suspect unless a large gap occurs.

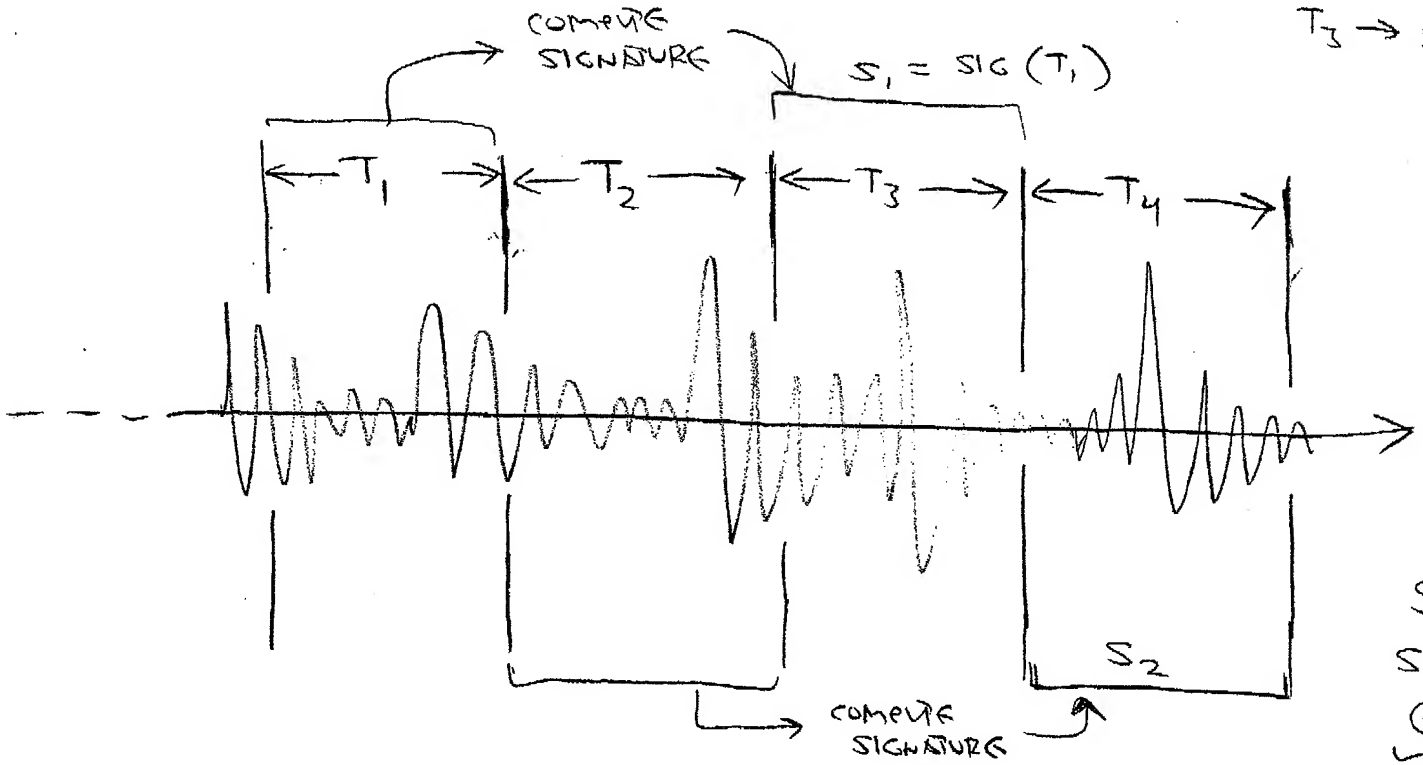
# # #



T_1 → SIG → S_1, TX IN T_3

T_2 → SIG → S_2, transmit IN T_4

T_3 → SIG → S_3, IN T_5

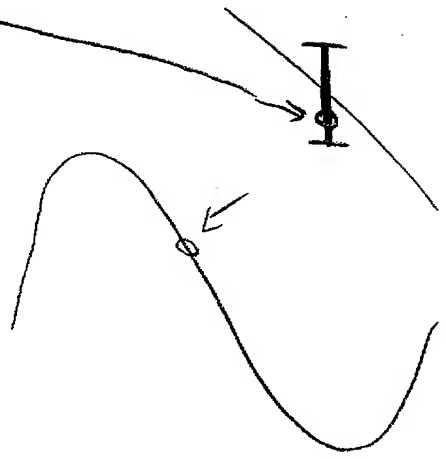


S_2 = SIG (T_2) ⊕ S_1 (optional)

* SIGNATURE should be compact (ECC)

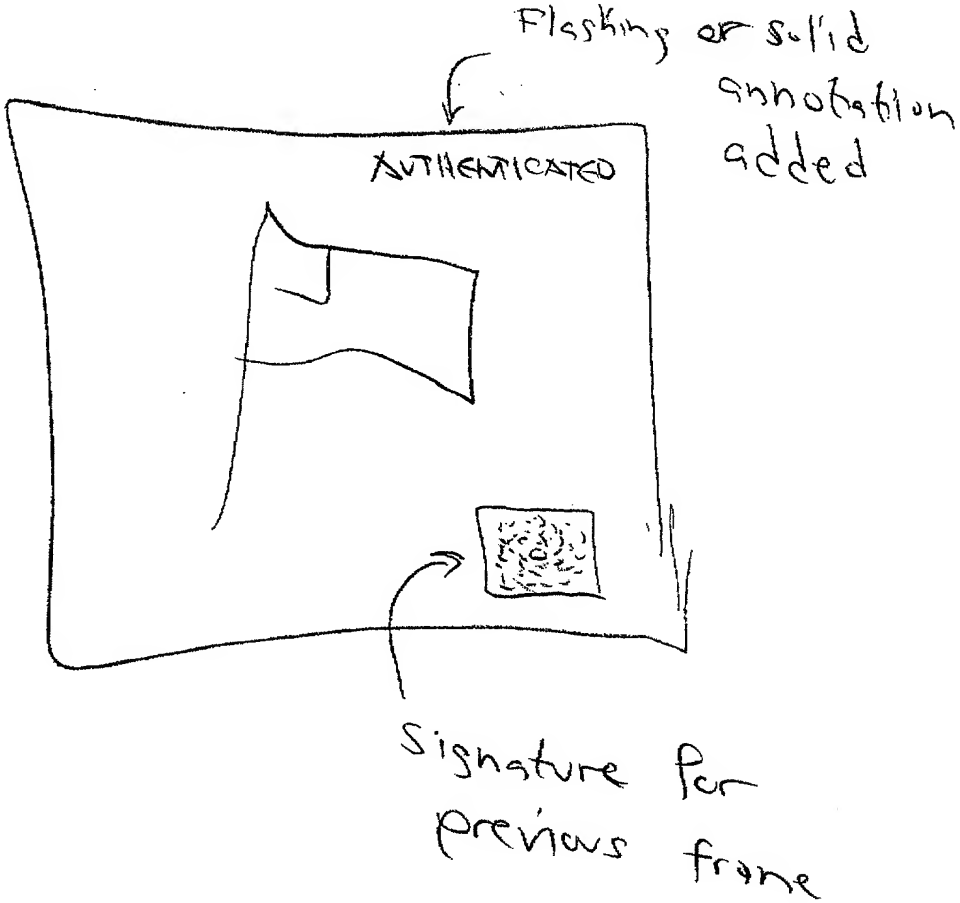
* Can calculate signature based on truncated sample words

Should be set to nearest value* in truncated wordlength to allow for some noise.



* 256-bit signature (just a guess)
10-second ~~blocks~~ frames
= 25 BPS ← can be subaudible or DSSS

* Can put signature barcode (2D) in video image. Will be blanked out in calculation of that ~~block's~~ signature frame



Multimedia Data-Embedding and Watermarking Technologies

MITCHELL D. SWANSON, MEMBER, IEEE, MEI KOBAYASHI, AND AHMED H. TEWFIK, FELLOW, IEEE

Invited Paper

In this paper, we review recent developments in transparent data embedding and watermarking for audio, image, and video. Data-embedding and watermarking algorithms embed text, binary streams, audio, image, or video in a host audio, image, or video signal. The embedded data are perceptually inaudible or invisible to maintain the quality of the source data. The embedded data can add features to the host multimedia signal, e.g., multilingual soundtracks in a movie, or provide copyright protection. We discuss the reliability of data-embedding procedures and their ability to deliver new services such as viewing a movie in a given rated version from a single multicast stream. We also discuss the issues and problems associated with copy and copyright protections and assess the viability of current watermarking algorithms as a means for protecting copyrighted data.

***Keywords*—Copyright protection, data embedding, steganography, watermarking.**

I. Introduction

The past few years have seen an explosion in the use of digital media. Industry is making significant investments to deliver digital audio, image, and video information to consumers and customers. A new infrastructure of digital audio, image, and video recorders and players, on-line services, and electronic commerce is rapidly being deployed. At the same time, major corporations are converting their audio, image, and video archives to an electronic form.

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Digital media offer several distinct advantages over analog media: the quality of digital audio, image, and video signals is higher than that of their analog counterparts. Editing is easy because one can access the exact discrete locations that should be changed. Copying is simple with no loss of fidelity. A copy of a digital media is identical to the original. Digital audio, image, and videos are easily transmitted over networked information systems.

These advantages have opened up many new possibilities. In particular, it is possible to hide data (information) within digital audio, image, and video files. The information is hidden in the sense that it is perceptually and statistically undetectable. With many schemes, the hidden information can still be recovered if the host signal is compressed, edited, or converted from digital to analog format and back.

As we shall see in Section II, pure analog data-hiding techniques had been developed in the past. However, these techniques are not as robust as most of the digital data hiding techniques that we review in this paper. Furthermore, they cannot embed as much data in a host signal as the digital approaches.

Digital data embedding has many applications. Foremost is passive and active copyright protection. Many of the inherent advantages of digital signals increase problems associated with copyright enforcement. For this reason, creators and distributors of digital data are hesitant to provide access to their intellectual property. Digital watermarking has been proposed as a means to identify the owner or distributor of digital data.

Data embedding also provides a mechanism for embedding important control, descriptive, or reference information in a given signal. This information can be used for tracking the use of a particular clip, e.g., for pay-per-use applications, including billing for commercials and video and audio broadcast, as well as Internet electronic commerce of digital media. It can be used to track audio or visual object creation, manipulation, and modification history within a given signal without the overhead associated with creating

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he knows that the host signal contains data and is familiar with the exact algorithm for embedding the data. Note that in some applications, e.g., covert communications, the data may also be encrypted prior to insertion in a host signal.

F. Copyright Protection and Ownership Deadlock

Data-embedding algorithms may be used to establish ownership and distribution of data. In fact, this is the application of data embedding or watermarking that has received most attention in the literature. Unfortunately, most current watermarking schemes are unable to resolve rightful ownership of digital data when multiple ownership claims are made, i.e., when a deadlock problem arises. The inability of many data-embedding algorithms to deal with deadlock, first described by Craver *et al.* [15], is independent of how the watermark is inserted in the multimedia data or how robust it is to various types of modifications.

Today, no scheme can unambiguously determine ownership of a given multimedia signal if it does not use an original or other copy in the detection process to at least construct the watermark to be detected. A pirate can simply add his watermark to the watermarked data or counterfeit a watermark that correlates well or is detected in the contested signal. Current data-embedding schemes used as copyright-protection algorithms are unable to establish who watermarked the data first. Furthermore, none of the current data-embedding schemes has been proven to be immune to counterfeiting watermarks that will correlate well with a given signal as long as the watermark is not restricted to depend partially in a noninvertible manner on the signal.

If the detection scheme can make use of the original to construct the watermark, then it may be possible to establish unambiguous ownership of the data regardless of whether the detection scheme subtracts the original from the signal under consideration prior to watermark detection or not. Specifically, [16] derives a set of sufficient conditions that watermarks and watermarking schemes must satisfy to provide unambiguous proof of ownership. For example, one can use watermarks derived from pseudorandom sequences that depend on the signal and the author. Reference [16] establishes that this will work for *all* watermarking procedures regardless of whether they subtract the original from the signal under consideration prior to watermark detection or not. Reference [85] independently derived a similar result for a restricted class of watermarking techniques that rely on subtracting a signal derived from the original from the signal under consideration prior to watermark detection. The signal-dependent key also helps to thwart the "mix-and-match" attack described in [16].

An author can construct a watermark that depends on the signal and the author and provides unambiguous proof of ownership as follows. The author has two random keys x_1 and x_2 (i.e., seeds) from which a pseudorandom sequence y can be generated using a suitable pseudorandom sequence generator [76]. Popular generators include RSA, Rabin, Blum/Micali, and Blum/Blum/Shub [25]. With the two proper keys, the watermark may be extracted. Without the two keys, the data hidden in the signal are statistically undetectable and impossible to recover. Note that classical maximal length pseudonoise sequences (i.e., m-sequence) generated by linear feedback shift registers are *not* used to generate a watermark. Sequences generated by shift registers are cryptographically insecure: one can solve for the feedback pattern (i.e., the keys) given a small number of output bits y.

The noise-like sequence y may be used to derive the actual watermark hidden into the signal or to control the operation of the watermarking algorithm, e.g., to determine the location of pixels that may be modified. The key x_1 is *author* dependent. The key x_2 is *signal* dependent. The key x_1 is the secret key assigned to (or chosen by) the author. The key x_2 is *computed from the signal* that the author wishes to watermark. It is computed from the signal using a one-way hash function. For example, the tolerable error levels supplied by masking models (see Section IV) are hashed in [85] to a key x_2. Any one of a number of well-known secure one-way hash functions may be used to compute x_2, including RSA, MD4 [77], and SHA [60]. For example, the Blum/Blum/Shub pseudorandom generator uses the one-way function $y = g_n(x) = x^2 \bmod n$, where $n = pq$ for primes p and q so that $p = q = 3 \bmod 4$. It can be shown that generating x or y from partial knowledge of y is *computationally infeasible* for the Blum/Blum/Shub generator.

The signal-dependent key x_2 makes counterfeiting very difficult. The pirate can only provide key x_1 to the arbitrator. Key x_2 is automatically computed by the watermarking algorithm from the original signal. As it is computationally infeasible to invert the one-way hash function, the pirate is unable to fabricate a counterfeit original that generates a desired or predetermined watermark.

Deadlock may also be resolved using the dual watermarking scheme of [85]. That scheme employs a *pair* of watermarks. One watermarking procedure requires the original data set for watermark detection. The second watermarking procedure does *not* require the original data set. A data-embedding technique that satisfies the restrictions outlined in [16] can be used to insert the second watermark.

The above discussion clearly highlights the limitation of watermarking as an unambiguous mean of establishing ownership. Future clever attacks may show that the schemes described in [16] or [85] are still vulnerable to deadlock. Furthermore, all parties would need to use watermarking techniques that have been proven or certified to be immune to deadlock to establish ownership of media. Note also that contentions of ownership can occur in too many different forms. Copyright protection will probably not be resolved exclusively by one group or even the entire technical community since it involves too many legal issues, including the very definition of similarity and derived works. Many multidisciplinary efforts are currently investigating standards and rules for national and international copyright protection and enforcement in the digital age.

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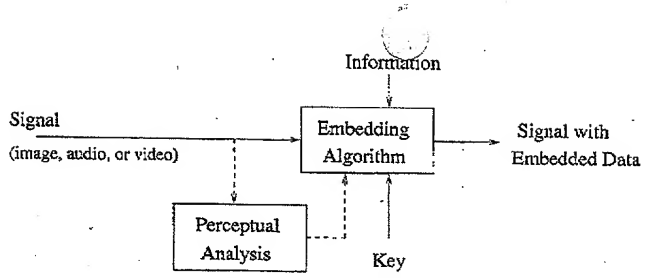

Fig. 1. Diagram of a data-embedding algorithm. The information is embedded into the signal using the embedding algorithm and a key. The dashed lines indicate that the algorithm may directly exploit perceptual analysis to embed information.

IV. Signal Insertion: The Role of Masking

The first problem that all data-embedding and watermarking schemes need to address is that of inserting data in the digital signal without deteriorating its perceptual quality. Of course, we must be able to retrieve the data from the edited host signal, i.e., the insertion method must also be invertible. Since the data-insertion and data-recovery procedures are intimately related, the insertion scheme must take into account the requirement of the data-embedding application. In many applications, we will need to be able to retrieve the data even when the host signal has undergone modifications, such as compression, editing, or translation between formats, including A/D and D/A conversions.

Data insertion is possible because the digital medium is ultimately consumed by a human. The human hearing and visual systems are imperfect detectors. Audio and visual signals must have a minimum intensity or contrast level before they can be detected by a human. These minimum levels depend on the spatial, temporal, and frequency characteristics of the human auditory and visual systems. Further, the human hearing and visual systems are characterized by an important phenomenon called masking. Masking refers to the fact that a component in a given audio or visual signal may become imperceptible in the presence of another signal called the masker. Most signal-coding techniques (e.g., [41]) exploit the characteristics of the human auditory and visual systems directly or indirectly. Likewise, all data-embedding techniques exploit the characteristics of the human auditory and visual systems implicitly or explicitly (see Fig. 1). In fact, embedding data would not be possible without the limitations of the human visual and auditory systems. For example, it is not possible to modify a binary stream that represents programs or numbers that will be interpreted by a computer. The modification would directly and adversely affect the output of the computer.

A. The Human Auditory System (HAS)

Audio masking is the effect by which a faint but audible sound becomes inaudible in the presence of another louder audible sound, i.e., the masker [42]. The masking effect depends on the spectral and temporal characteristics of both the masked signal and the masker.

Frequency masking refers to masking between frequency components in the audio signal. If two signals that occur simultaneously are close together in frequency, the stronger masking signal may make the weaker signal inaudible. The masking threshold of a masker depends on the frequency, sound pressure level, and tone-like or noise-like characteristics of both the masker and the masked signal [61]. It is easier for a broad-band noise to mask a tonal signal than for a tonal signal to mask out a broad-band noise. Moreover, higher frequency signals are more easily masked.

The human ear acts as a frequency analyzer and can detect sounds with frequencies that vary from 10 to 20 000 Hz. The HAS can be modeled by a set of bandpass filters with bandwidths that increase with increasing frequency. The bands are known as the critical bands. The critical bands are defined around a center frequency in which the noise bandwidth is increased until there is a just noticeable difference in the tone at the center frequency. Thus, if a faint tone lies in the critical band of a louder tone, the faint tone will not be perceptible.

Frequency-masking models are readily obtained from the current generation of high-quality audio codecs, e.g., the masking model defined in the International Standards Organization (ISO)-MPEG Audio Psychoacoustic Model 1 for Layer I [40]. The Layer I masking method is summarized as follows for a 32-kHz sampling rate. The MPEG model also supports sampling rates of 44.1 and 48 kHz.

The frequency mask is computed on localized segments (or windows) of the audio signal. The first step consists of computing the power spectrum of a short window (512 or 1024 samples) of the audio signal. Tonal (sinusoidal) and nontonal (noisy) components in the spectrum are identified because their masking models are different. A tonal component is a local maximum of the spectrum. The auditory system behaves as a bank of bandpass filters, with continuously overlapping center frequencies. These "auditory filters" can be approximated by rectangular filters with critical bandwidth increasing with frequency. In this model, the audible band is therefore divided into 24 nonregular critical bands.

Next, components below the absolute hearing threshold and tonal components separated by less than 0.5 Barks are removed. The final step consists of computing individual and global masking thresholds. The frequency axis is discretized according to hearing sensitivity and express frequencies in Barks. Note that hearing sensitivity is higher at low frequencies. The resulting masking curves are almost linear and depend on a masking index different for tonal and nontonal components. They are characterized by different lower and upper slopes depending on the distance between the masked and the masking component. We use f_1 to denote the set of frequencies present in the test signal. The global masking threshold for each frequency f_2 takes into account the absolute hearing threshold S_a and the masking curves P_2 of the N_t tonal components and N_n nontonal components

$$S_m(f_2) = 10 * \log_{10}\left[10^{S_a(f_2)/10} + \sum_{j=1}^{N_t} 10^{P_2(f_2,f_1,P_1)/10} + \sum_{j=1}^{N_n} 10^{P_2(f_2,f_1,P_1)/10}\right]. \quad (1)$$

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AFFIDAVIT OF JOHN Q. PUBLIC

John Q. Public declares as follows:

1. I have been presented with a ZIP file entitled FILES.ZIP containing, among other files, an executable file entitled HASH.EXE and a document in Adobe Acrobat format entitled DOCUMENT.PDF. After extracting HASH.EXE from FILES.ZIP and launching it, I opened FILES.ZIP in the program window that was created upon launch. I then saw a display appear in the program window with the legend "160-bit Hash Value (SHA1)" and a string of alphanumeric characters. The characters were identical to those printed below and repeated throughout the background of this entire paper as vertically oriented digits in various outline fonts.

BD7D F2FD EC1C DF14 4811 574F F7CE 7D1E 6EB6 F7E9

2. I extracted DOCUMENT.PDF from FILES.ZIP and printed out the document reproduced therein. I took the printed copy into the file inspection room of the U.S. Patent Office and compared each page printed from DOCUMENT.PDF with the papers physically mounted in the official file of U.S. Patent 6,052,748.

3. I personally confirmed that each page of DOCUMENT.PDF is a true and correct copy of a page in the file of U.S. Patent 6,052,748. I further confirmed that all pages of the file have been reproduced in DOCUMENT.PDF.

I declare under penalty of perjury that the foregoing statements are true and correct.

Signed at _____, _____ This _____ day of _____, _____
City *State* *Date*

By: VOID
John Q. Public

This _____ day of _____, _____, the above-named person personally came before me and executed the foregoing Affidavit in my presence.

Notary: Outline digits are in background of entire document. Please stamp over digits in space above.

Notary Public: VOID

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AFFIDAVIT OF JAMES T. GRIFFIN

James T.____ declares as follows:

1. I have been presented with a ZIP file entitled 1033.ZIP containing, among other files, another ZIP file entitled 1033-SRC.ZIP, which contains an executable program file entitled GPG.EXE and a document in Adobe Acrobat format entitled 1033-ART.PDF. After extracting GPG.EXE from 1033-SRC.ZIP, I executed it from the Windows 98/NT/2000 command line with the following text:

```
gpg --print-md sha1 1033.zip
```

I then saw a display appear in the program window with a string of alphanumeric characters. The characters were identical to those printed below this paragraph and repeated throughout the background of this entire paper as vertically oriented digits in various outline fonts.

8D06 7D30 7CDD 969E 3C3C C57D E9E3 467D 2E54 6AC3

2. I opened 1033-ART.PDF, which was also extracted from 1033.ZIP, and viewed it on my computer screen. I carefully compared the displayed document with an original artwork having images of flowers formed from torn-out pieces of paper, fixed to a piece of construction paper beneath a transparent laminating surface.

3. I personally confirmed that 1033-ART.PDF is a true and correct reproduction of the original artwork, to the degree of accuracy permitted by the displayed reproduction. The only visible discrepancies between the reproduction and the original artwork are (1) the coloration of the background, which appears as a washed-out green in the original artwork and has more of a purplish hue in the reproduction, and (2) the lower 1 inch or so of the original artwork is not shown in the reproduction.

I declare under penalty of perjury that the foregoing statements are true and correct.

Signed at _____, _____ This _____ day of _____, _____
City *State* *Date*

By: VOID
James T.____

This _____ day of _____, _____, the above-named person personally came before me and executed the foregoing Affidavit in my presence.

Notary: Outline digits are in background of entire document. Please stamp over digits in space above.

Notary Public: VOID

APPENDIX H

FILING CONTENTS AUTHENTICATION

Joe Agent creates a document when filing a patent application for a client. He wants an additional piece of evidence that the patent application was mailed and that what was mailed was the patent application. So, he creates a PDF file of the application's text and drawings and computes the SHA1 hash of the PDF file using some the free GPG software ("GNU Privacy

Guard"), which he trusts. He makes a copy of the Express Mail label under which the application is to be mailed BEFORE having the label initialed and dated by the postal worker.

He then makes a Word 97 document that includes a brief block of text for a witness to sign and date. The block of text includes the Express Mail label and the hash. The document displays the digits of the hash as vertically oriented digits in various outline fonts, as

with the ACI (See, e.g., Appendix A.) He then puts the photocopy of the non-initialed express

mail label in his printer's paper supply, and prints the Word 97 (RTM) document.

The resulting document bears the image of the non-initialed express mail label with the hash of the patent application PDF file throughout its background and in a block of text of the top. Joe presents the document to Jane Attorney, an attorney in the office next door, for her dated signature. Now he can present the PDF file and the signed and dated paper document as evidence that the patent application reproduced in the PDF file existed before the express mail label was used to mail a package. This provides evidence that the patent application, reproduced in the PDF file, was actually what was mailed on the date of the express mail label. The proof can be made a bit

stronger by including the express mail label number in a footer on the first page of the patent application, so that the PDF file contains the express mail label number shown in the document. But that's going way beyond the level of evidence currently needed to reconstruct a file lost by the PTO.

This particular embodiment of the inventions can be employed any situation where proof of mailing of a particular document is desired, without the need for digital signatures and the associated hassles of public key authentication. The witness to this document doesn't even need to know what digital signatures or hash codes are. He or she is simply testifying as to the existence of the document with that particular code.

$$f(x,y,N)=\left(\frac{x-y}{x}\right)^N$$

$$f(x,y,N)=\exp\left(N\cdot\ln\left(\left|1-\frac{1}{x}\cdot y\right|\right)\right)\cdot\cos\left[N\cdot\left(\frac{1}{2}-\frac{1}{2}\cdot 1\right)\cdot\pi\right]\ldots$$
$$+i\cdot\exp\left(N\cdot\ln\left(\left|1-\frac{1}{x}\cdot y\right|\right)\right)\cdot\sin\left[N\cdot\left(\frac{1}{2}-\frac{1}{2}\cdot 1\right)\cdot\pi\right]$$

$$f(x,y,N)=\exp\left(N\cdot\ln\left(\left|1-\frac{1}{x}\cdot y\right|\right)\right)\cdot 1\ldots$$
$$+i\cdot\exp\left(N\cdot\ln\left(\left|1-\frac{1}{x}\cdot y\right|\right)\right)\cdot\sin\left[N\cdot\left(\frac{1}{2}-\frac{1}{2}\cdot 1\right)\cdot\pi\right]$$

$$f(x,y,N):=\exp\left(N\cdot\ln\left(\left|1-\frac{1}{x}\cdot y\right|\right)\right)$$

M

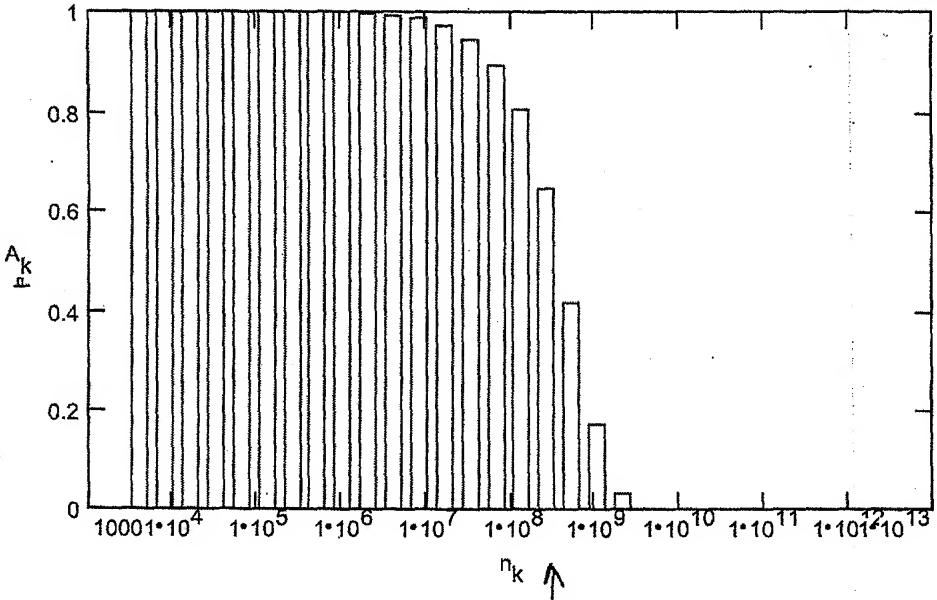
$x := 2^{32}$ $y := 7$ ← $p = 2^{32}-7$

$k := 12..40$ $n_k := 2^k$

$A_k := f(x,y,n_k)$

Graph: Product of probabilities of obtaining pass-through value, for each iteration, given n iterations.

output changed, i.e., not a



50% chance of getting one pass-through value w/ random inputs after 10^8 - 10^9 iterations.

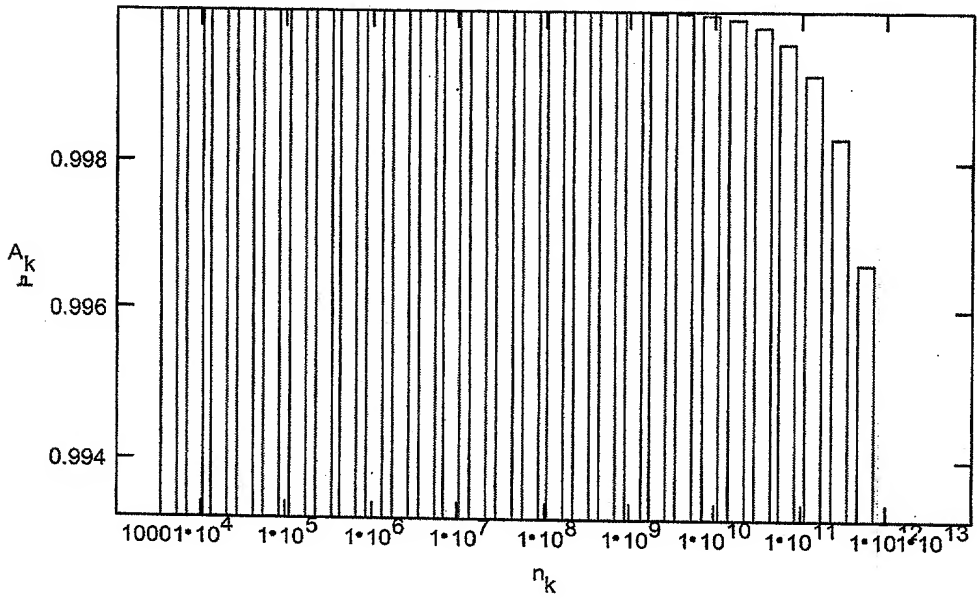
I-1

$x := 2^{50}$ ← Larger M

$A_k := f(x, y, n_k)$ Same k → changed output, i.e., not q

Graph: Product of probabilities of obtaining pass-through value for each iteration, given n iterations.

p = M − k

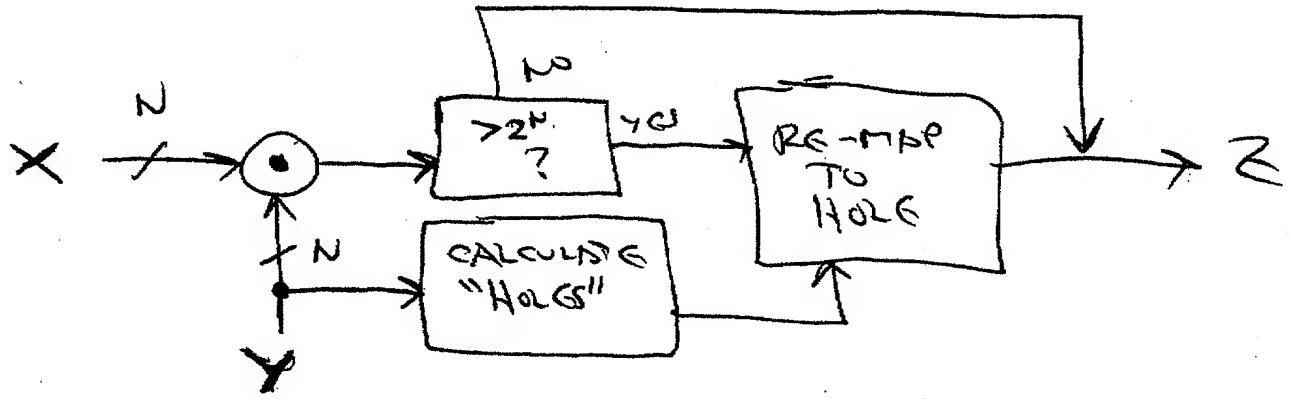


Need for exception handling can be made vanishingly unlikely.

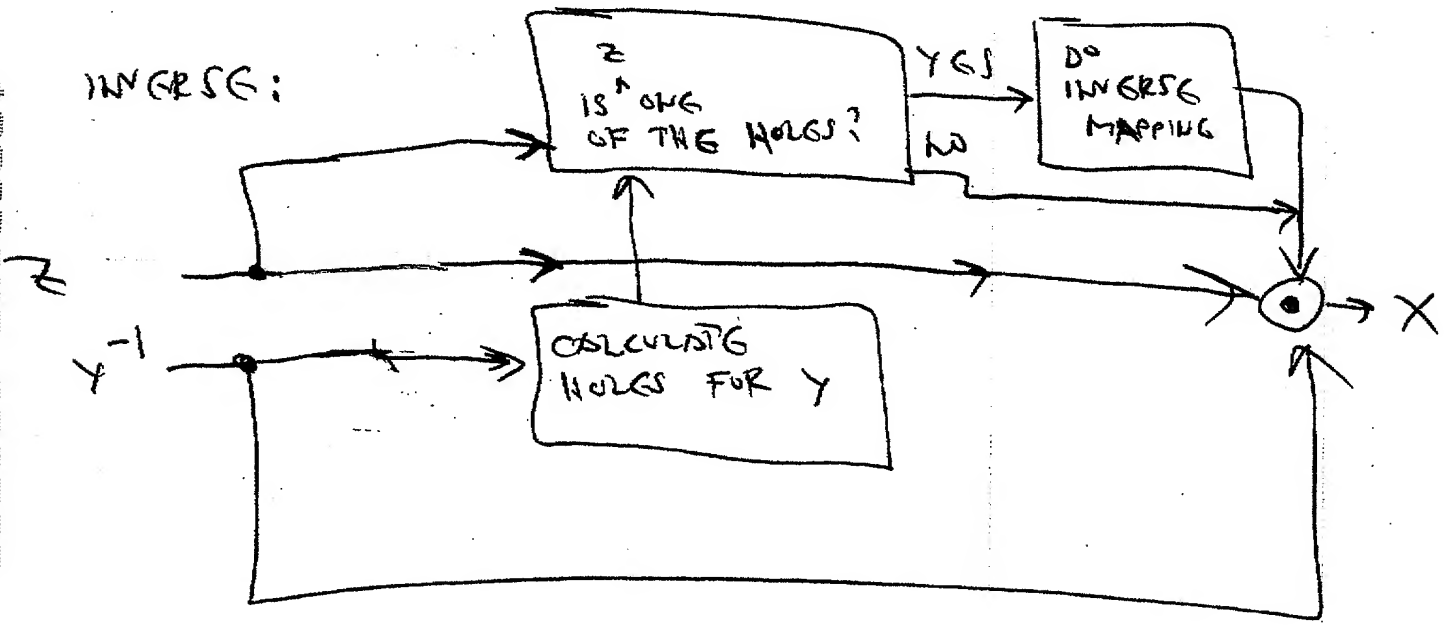
With $p > 2^N$:

MODIFIED "⊙"

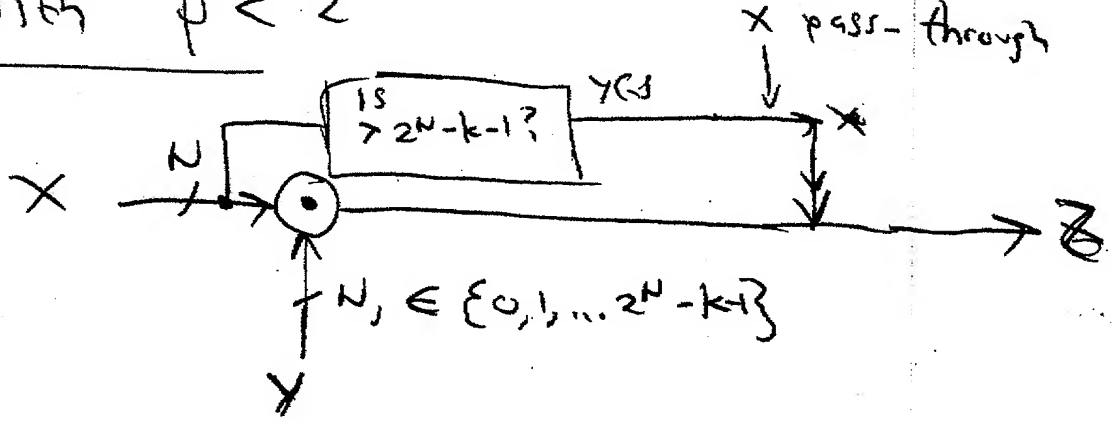
TO LOOK OUTSIDE $N = \{4, 8, 16\}$, $p = 2^N + 1$



INVERSE:



With $p < 2^N$



Interesting Primes (closest to 2^N)

$2^{20}+7$ $\quad$ $20^{20}-3$

$2^{32}+15$ $\quad$ $2^{32}-5$

2^N+1 not prime for $16 < N \leq 32$

$2^{16}+3$, -15

$2^{18}+3$, -5

$2^{19}-1$

$2^{28}+3$

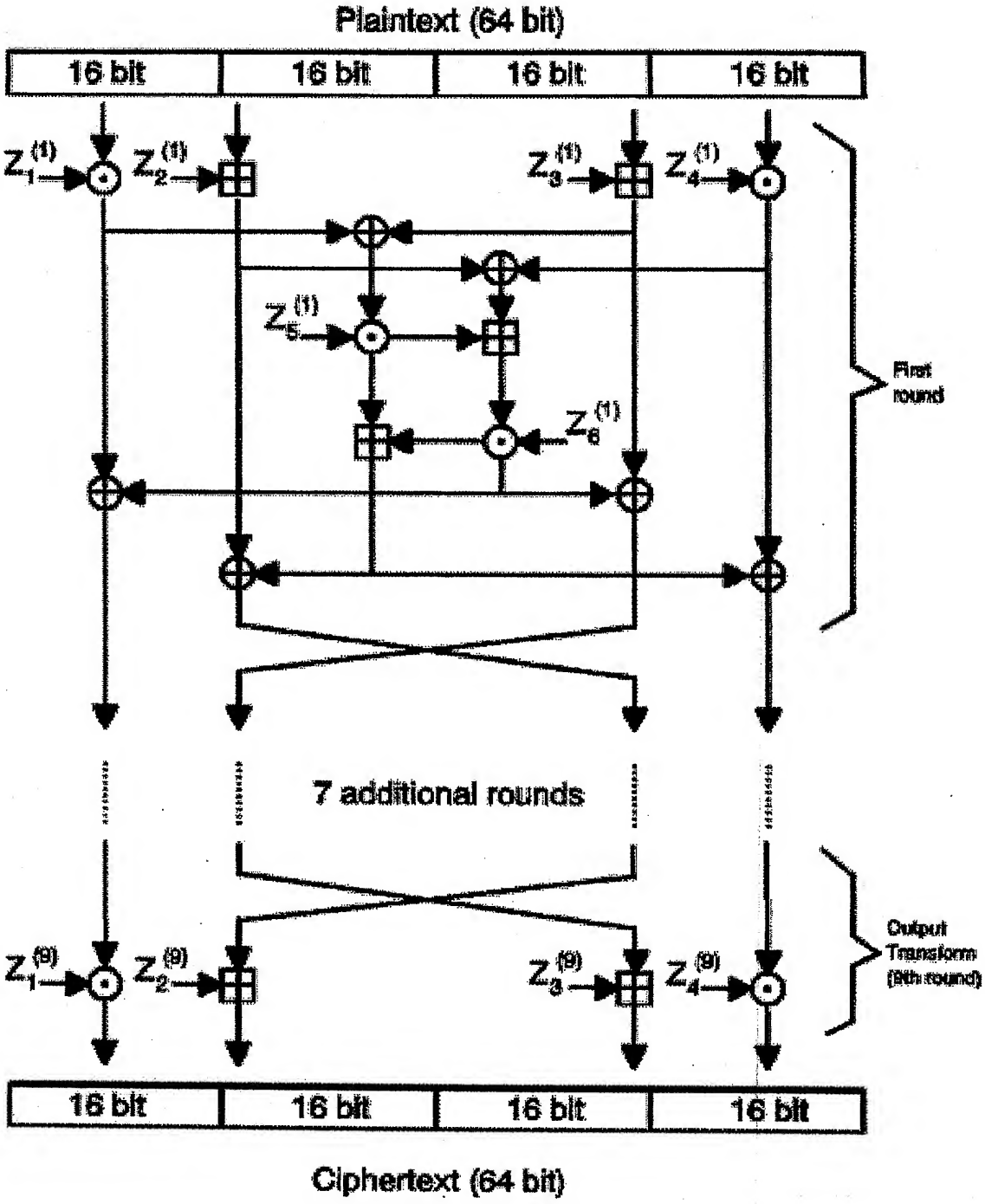
Bit Length	Test Number (Y=prime, N=not prime, blank=not tested)								
N	2^N-1	2^N-3	2^N-5	2^N-7	2^N-9	2^N-11	2^N-13	2^N-15	2^N-17
16	N	N	N	N	N	N	N	Y	
17	Y								
18	N	N							
19	Y								
20	N	Y							
21	N	N							
22	N	Y							
23	N	N							
24	N	Y							
25	N	N							
26	N	N							
27	N	N							
28	N	N							
29	N	Y							
30	N	N							
31	Y								
32	N	N	Y						
33	N	N	N	N	N				
34	N	N	N	N	N				
35	N	N	N	N	N				
36	N	N	Y						
37	N	N	N	N	N				
38	N	N	N	N	N				
39	N	N	N	Y					
40	N	N	N	N	N	N	N	N	N
41	N	N	N	N	N				
42	N	N	N	N	N				
43	N	N	N	N	N				
44	N	N	N	N	N				
45	N	N	N	N	N				
46	N	N	N	N	N				
47	N	N	N	N	N				
48	N	N	N	N	N	N	N	N	N

Effective bit length for one "leaked" bit, per offset below 2^N									
2^N	1	3	5	7	9	11	13	15	17
65536								12.0	
131072	16.0								
262144									
524288	18.0								
1048576		18.0							
2097152									
4194304		20.0							
8388608									
16777216		22.0							
33554432									
67108864									
134217728									
268435456									
536870912		27.0							
1073741824									
2147483648	30.0								
4294967296			29.4						
8589934592									
17179869184									
34359738368									
68719476736			33.4						
1.37439E+11									
2.74878E+11									
5.49756E+11				36.0					
1.09951E+12									
2.19902E+12									
4.39805E+12									
8.79609E+12									
1.75922E+13									
3.51844E+13									
7.03687E+13									
1.40737E+14									
2.81475E+14									

Likelihood of pass-through is $6/2^{32} = 1.4 \times 10^{-9}$ (very infrequent!)

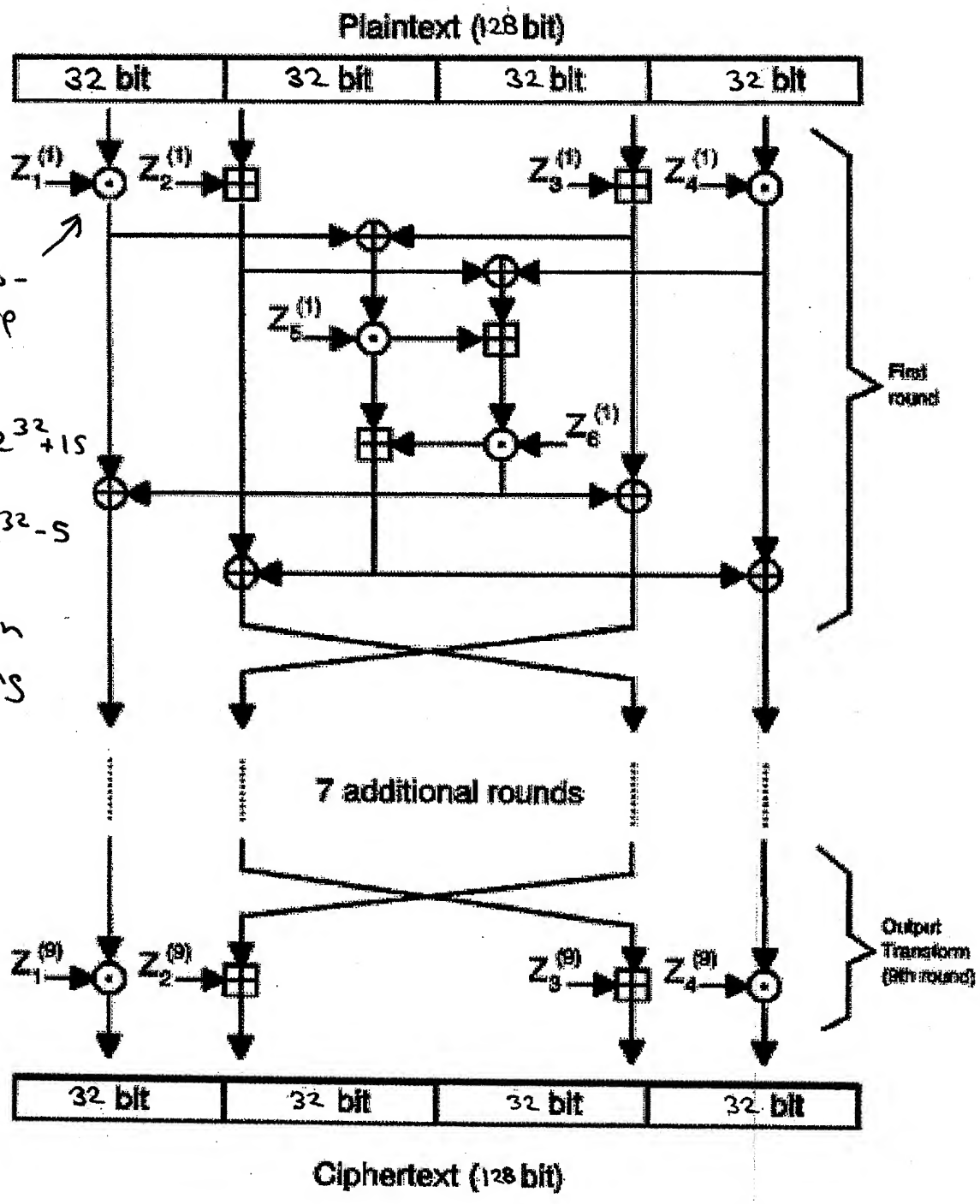
Six ~~five~~ members of $\{0,1\}^{32}$ ($4,294,967,290$; $4\ldots291$; $4\ldots292$; $4\ldots293$; $4\ldots294$; $4\ldots295$) are passed through function w/o modification.

4-3



IDEA CIPHER

From H.A.C. (Handbook of Applied Cryptography)



Adapted from H.A.C.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	2	4	6	8	10	12	1	3	5	7	9	11	13	14	15	16
3	3	6	9	12	2	5	8	11	1	4	7	10	13	14	15	16
4	4	8	12	3	7	11	2	6	10	1	5	9	13	14	15	16
5	5	10	2	7	12	4	9	1	6	11	3	8	13	14	15	16
6	6	12	5	11	4	10	3	9	2	8	1	7	13	14	15	16
7	7	1	8	2	9	3	10	4	11	5	12	6	13	14	15	16
8	8	3	11	6	1	9	4	12	7	2	10	5	13	14	15	16
9	9	5	1	10	6	2	11	7	3	12	8	4	13	14	15	16
10	10	7	4	1	11	8	5	2	12	9	6	3	13	14	15	16
11	11	9	7	5	3	1	12	10	8	6	4	2	13	14	15	16
12	12	11	10	9	8	7	6	5	4	3	2	1	13	14	15	16
13	ILLEGAL KEY VALUES															
14																
15																
16																

$N = 4, \quad p = 2^N - 3$

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	1	3	5	7	8
3	3	6	2	5	1	4	7	8
4	4	1	5	2	6	3	7	8
5	5	3	1	6	4	2	7	8
6	6	5	4	3	2	1	7	8
7	ILLEGAL KEY VALUES							
8								

$N = 3, \quad p = 2^N - 1$

	1	2	3	...	20,537	20,538	20,539	...	16,777,211	16,777,212	16,777,213	16,777,214	16,777,215	16,777,216
1	1	2	3	...	20,537	20,538	20,539	...	16,777,211	16,777,212	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
2	2	4	6	...	41,074	41,076	41,078	...	16,777,209	16,777,211	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
3	3	6	9	...	61,611	61,614	61,617	...	16,777,207	16,777,210	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
⋮	⋮	⋮	⋮		⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
20,537	20,537	41,074	61,611	...	2,338,044	2,358,581	2,379,118	...	16,736,139	16,756,676	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
20,538	20,538	41,076	61,614	...	2,358,581	2,379,119	2,399,657	...	16,736,137	16,756,675	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
20,539	20,539	41,078	61,617	...	2,379,118	2,399,657	2,420,196	...	16,736,135	16,756,674	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
⋮	⋮	⋮	⋮		⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
16,777,211	16,777,211	16,777,209	16,777,207	...	16,736,139	16,736,137	16,736,135	...	4	2	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
16,777,212	16,777,212	16,777,211	16,777,210	...	16,756,676	16,756,675	16,756,674	...	2	1	*16,777,213*	*16,777,214*	*16,777,215*	*16,777,216*
16,777,213	ILLEGAL KEY VALUES													
16,777,214														
16,777,215														
16,777,216														

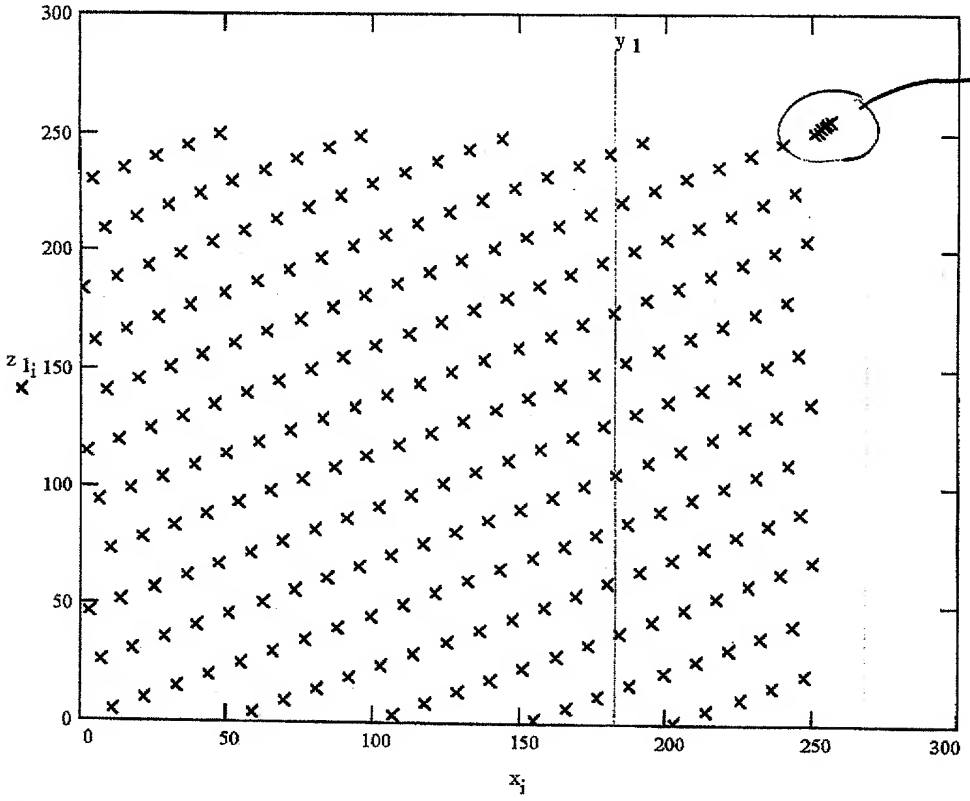
$N = 24, \quad \varphi = 2^N - 3$

4
1-7

Multiplicative Group Operation (pseudogroup)
f(x,y) = xy mod 2^N-k, where x,y ^N-k;
= x where x < 2^N-k. (y always < 2^N-k.)

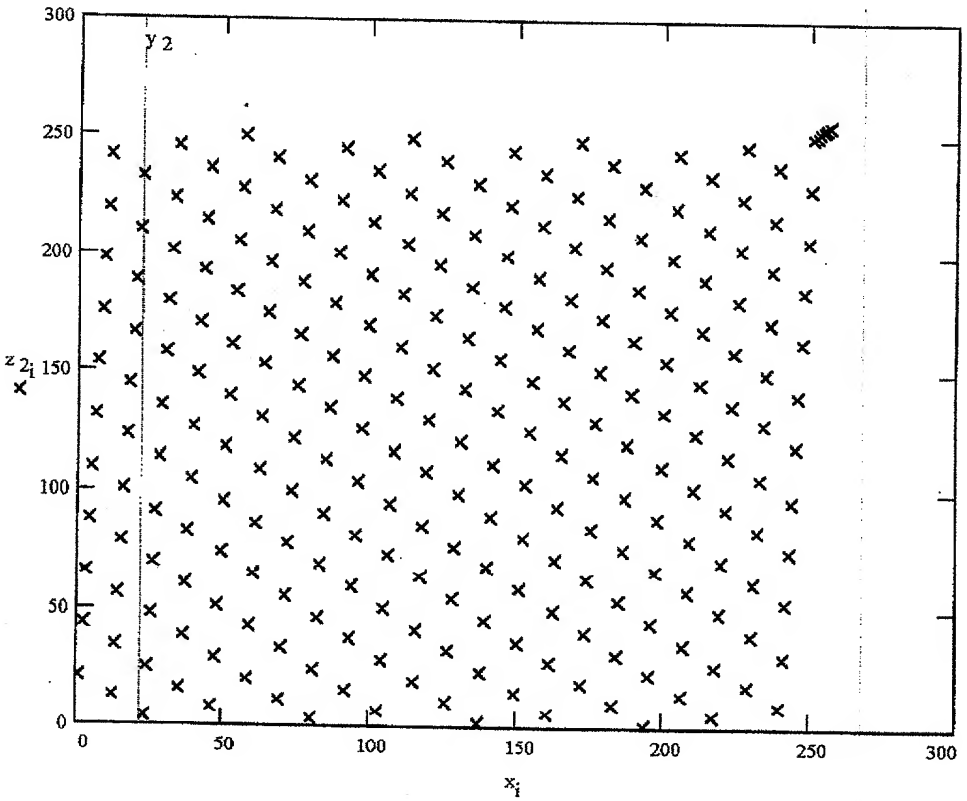
$N = 8$ bits $2^N = 256$ $k = 5$ $p = 251$ prime modulus

Pseudogroup operation with key y1: $y_1 = 183$ $z1_i := f(x_i, y_1)$



Linear exception region per EBS 'invention' Where x > p-1

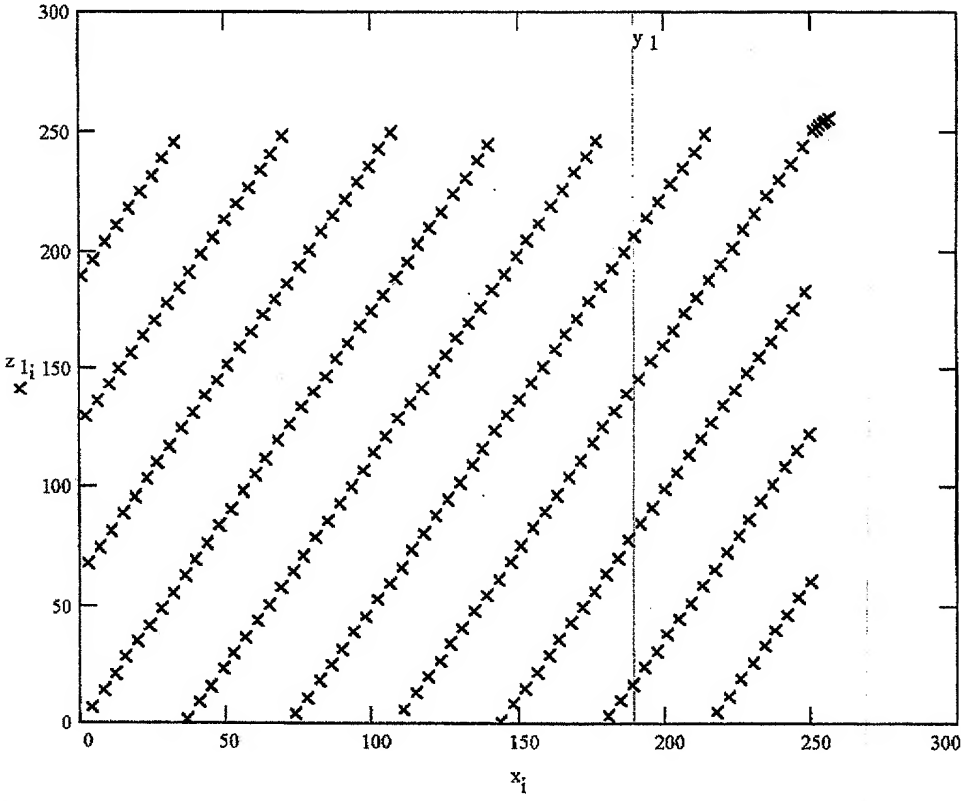
Psedogroup operation with key y2: $y_2 = 22$ $z2_i := f(x_i, y_2)$



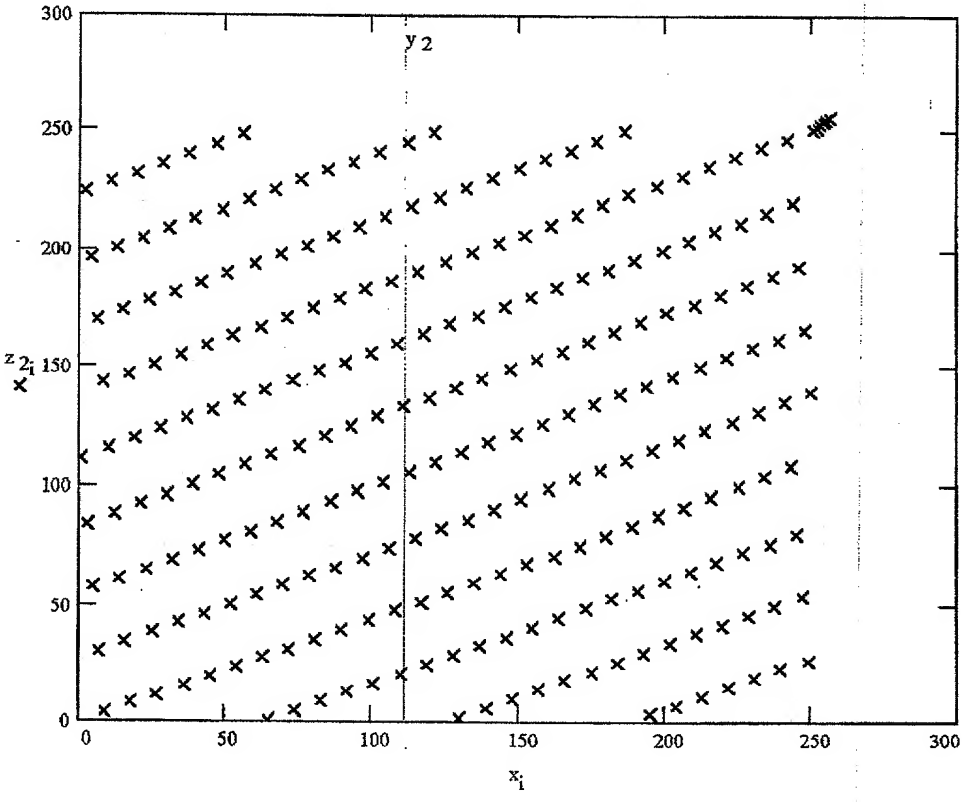
Multiplicative Group Op[illegible]ion (pseudogroup)
f(x,y) = xy mod 2^N-k, where x,y ^N-k;
= x where x < 2^N-k. (y always < 2^N-k.)

$N = 8$ bits $2^N = 256$ $k = 5$ $p = 251$ prime modulus

Pseudogroup operation with key y1: $y_1 = 190$ $z1_i := f(x_i, y_1)$



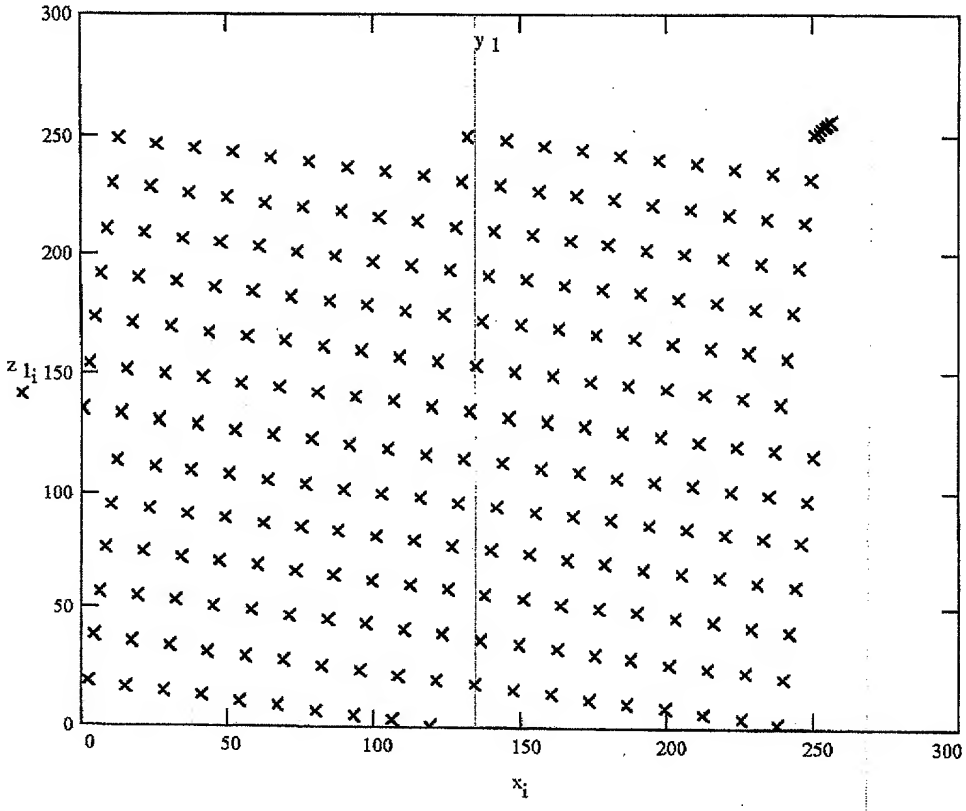
Pseudogroup operation with key y2: $y_2 = 112$ $z2_i := f(x_i, y_2)$



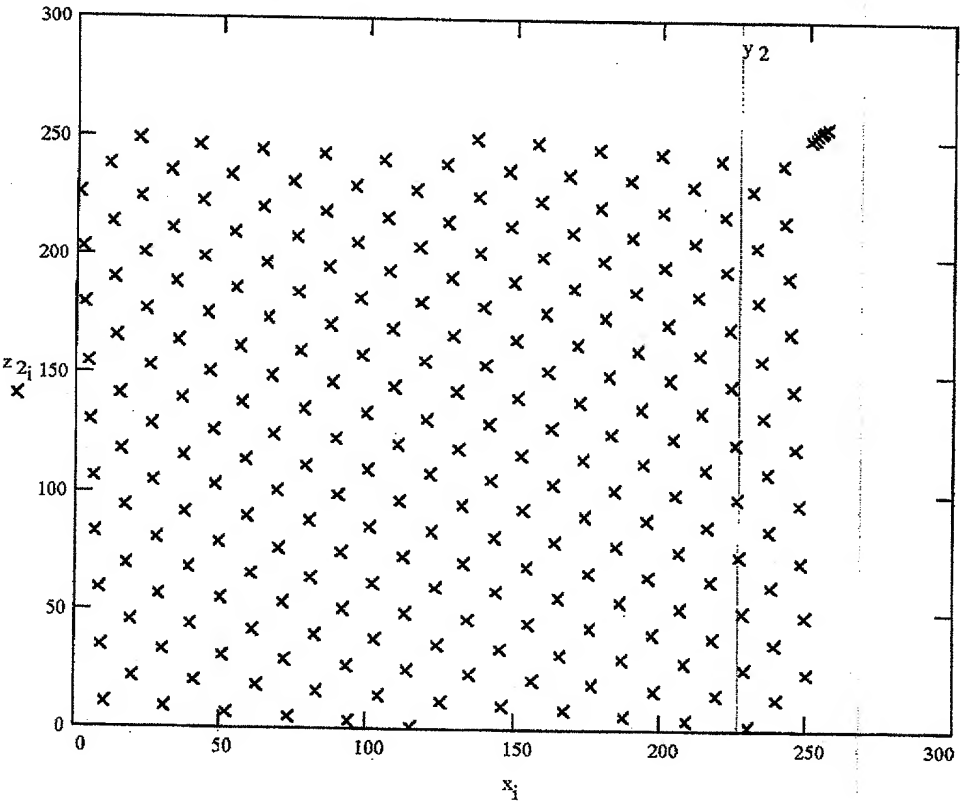
Multiplicative Group Op[illegible]ion (pseudogroup)
f(x,y) = xy mod 2^N-k, where x,y ^N-k;
= x where x < 2^N-k. (y always < 2^N-k.)

$N = 8$ bits $2^N = 256$ $k = 5$ $p = 251$ prime modulus

pseudogroup operation with key y1: $y_1 = 135$ $z1_i := f(x_i, y_1)$



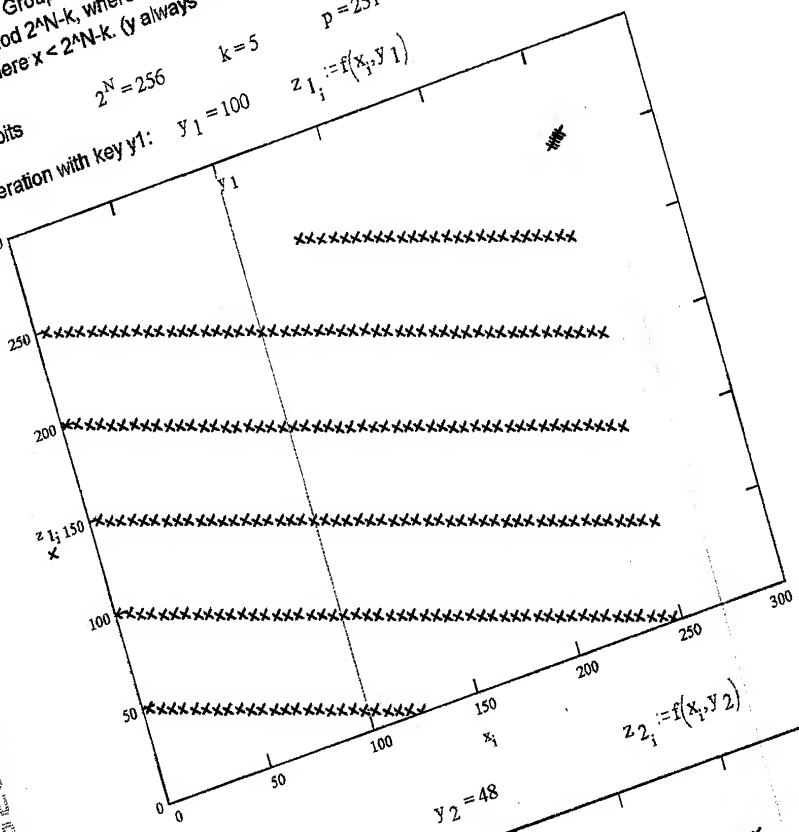
Pseudogroup operation with key y2: $y_2 = 227$ $z2_i := f(x_i, y_2)$



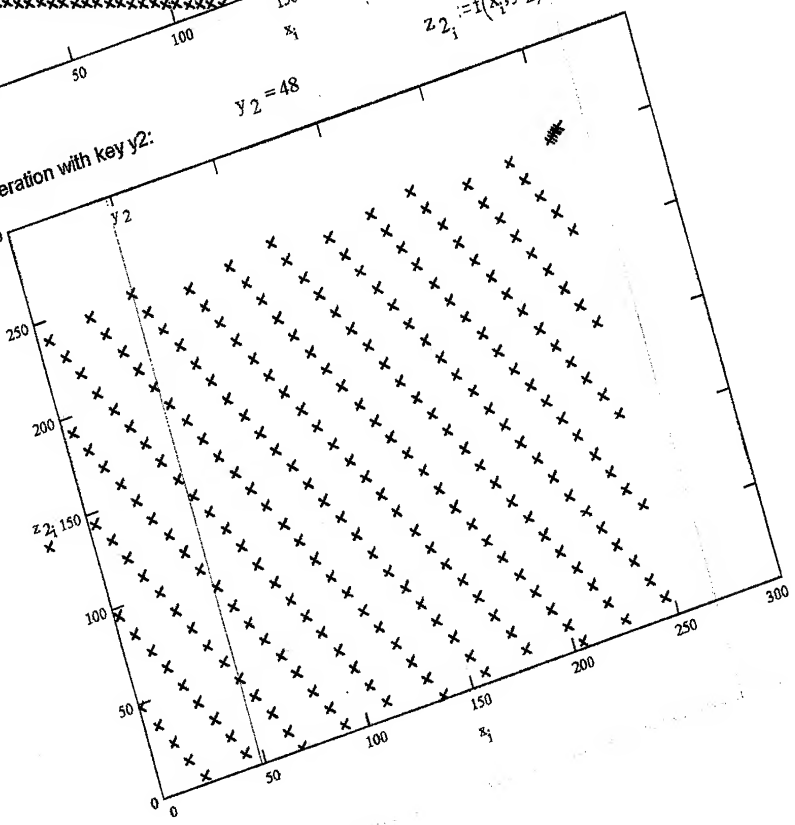
Multiplicative Group Operation (pseudogroup)
f(x,y) = xy mod 2^N-k, where x,y ^N-k;
= x where x < 2^N-k. (y always < 2^N-k.)

$N = 8$ bits $\quad 2^N = 256 \quad k = 5 \quad p = 251$ prime modulus

Pseudogroupoperation with key y1: $y_1 = 100$ $\quad z1_i := f(x_i, y_1)$



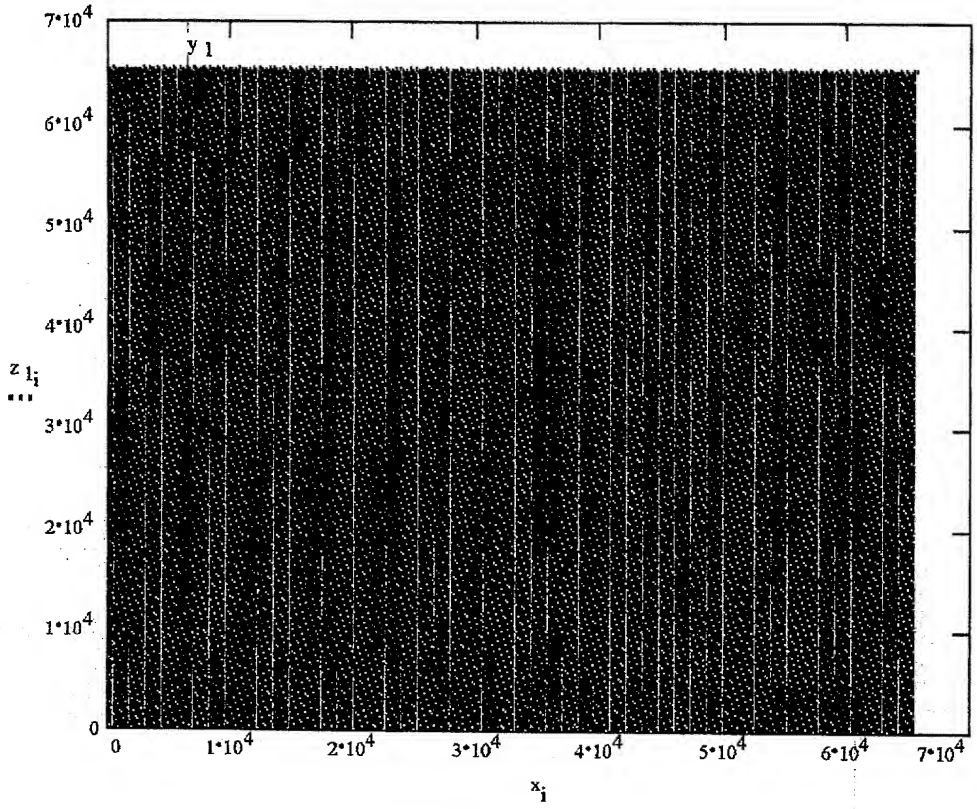
Pseudogroupoperation with key y2: $y_2 = 48$ $\quad z2_i := f(x_i, y_2)$



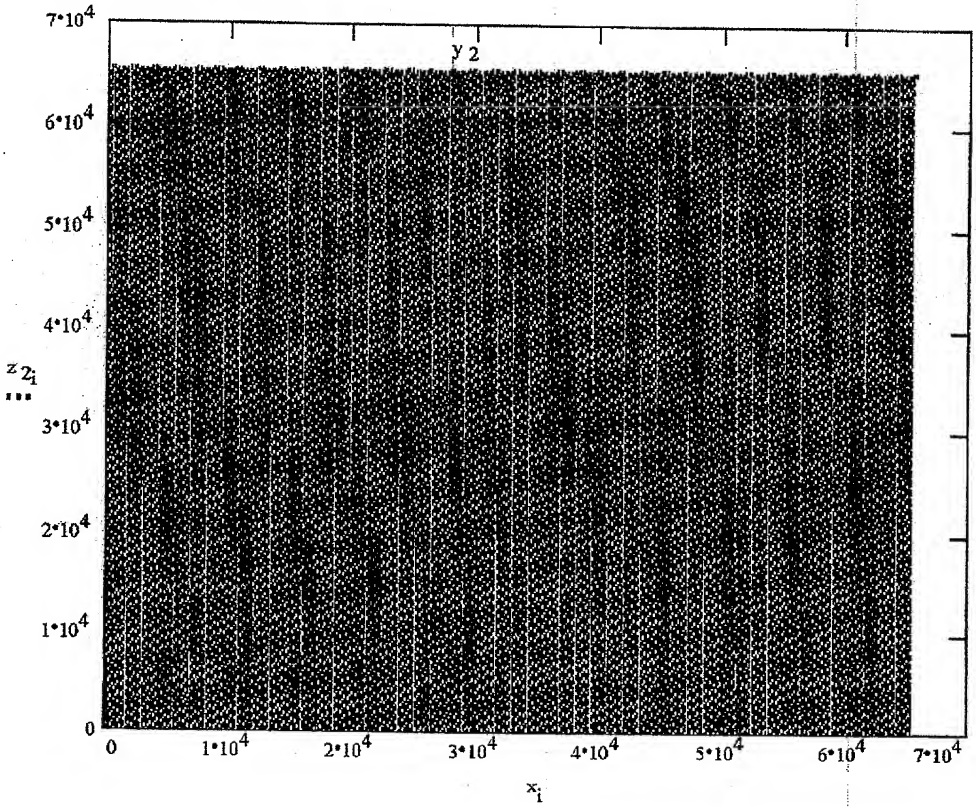
Multiplicative Group Operation (pseudogroup)
f(x,y) = xy mod 2^N-k, where x,y ^N-k;
= x where x < 2^N-k. (y always < 2^N-k.)

$N = 16$ bits $2^N = 65536$ $k = 15$ $p = 65521$ prime modulus

pseudogroup operation with key y1: $y_1 = 6595$ $z1_i := f(x_i, y_1)$

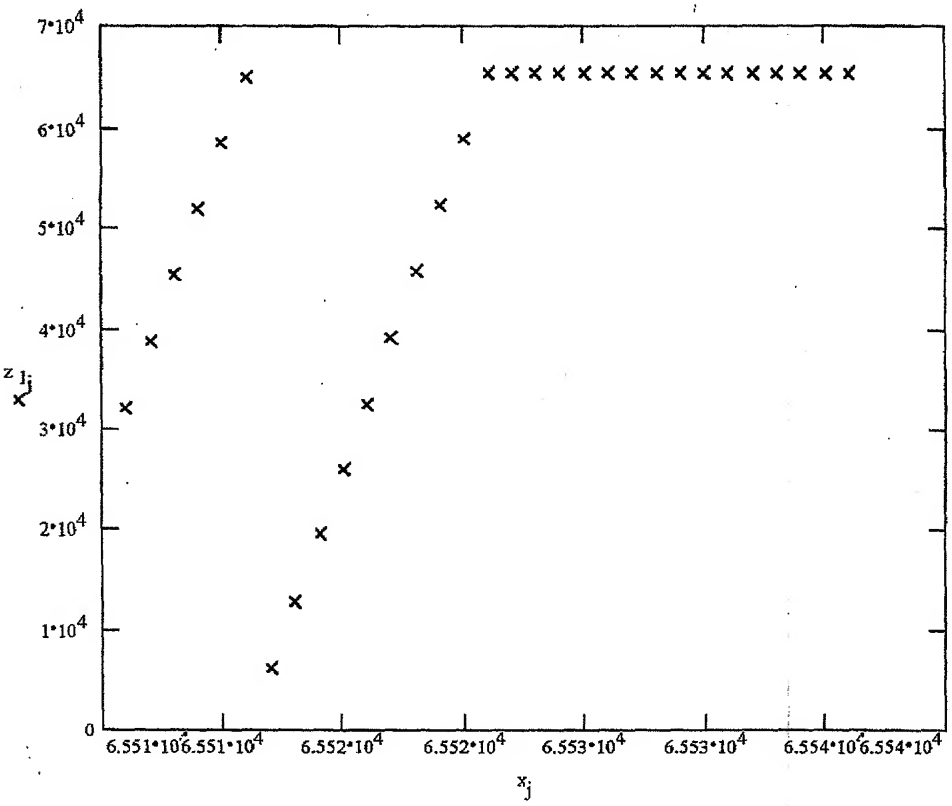


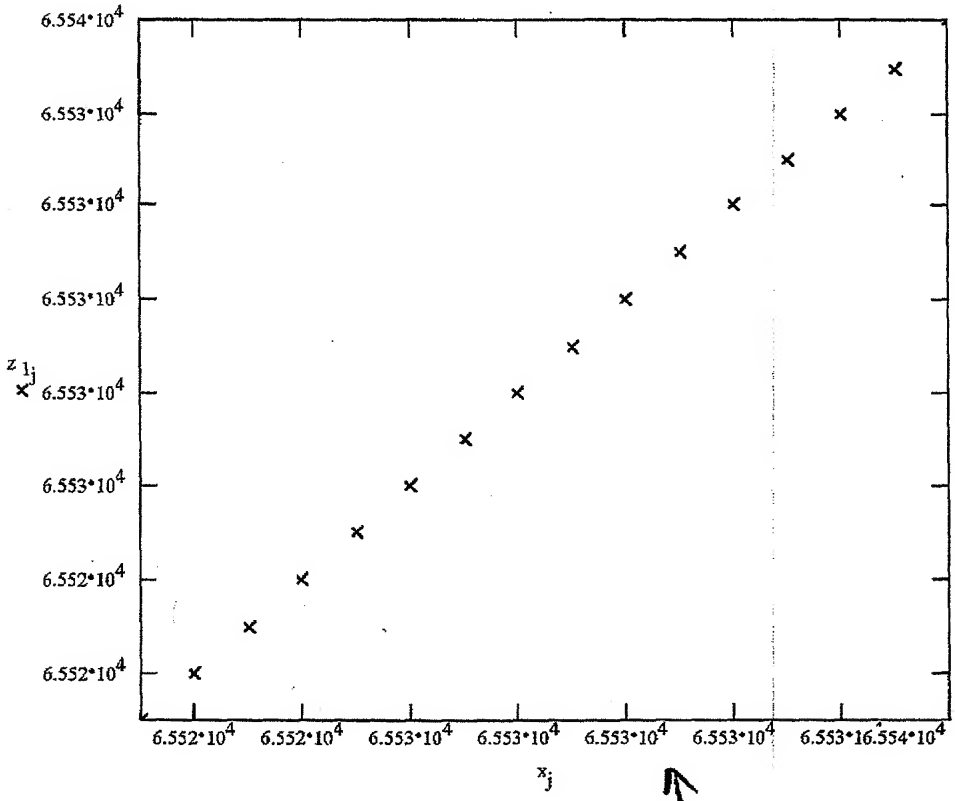
pseudogroup operation with key y2: $y_2 = 27939$ $z2_i := f(x_i, y_2)$



Zoom in on exception region of plot with key Y1

$j := 2^N - 2 \cdot k .. 2^N$





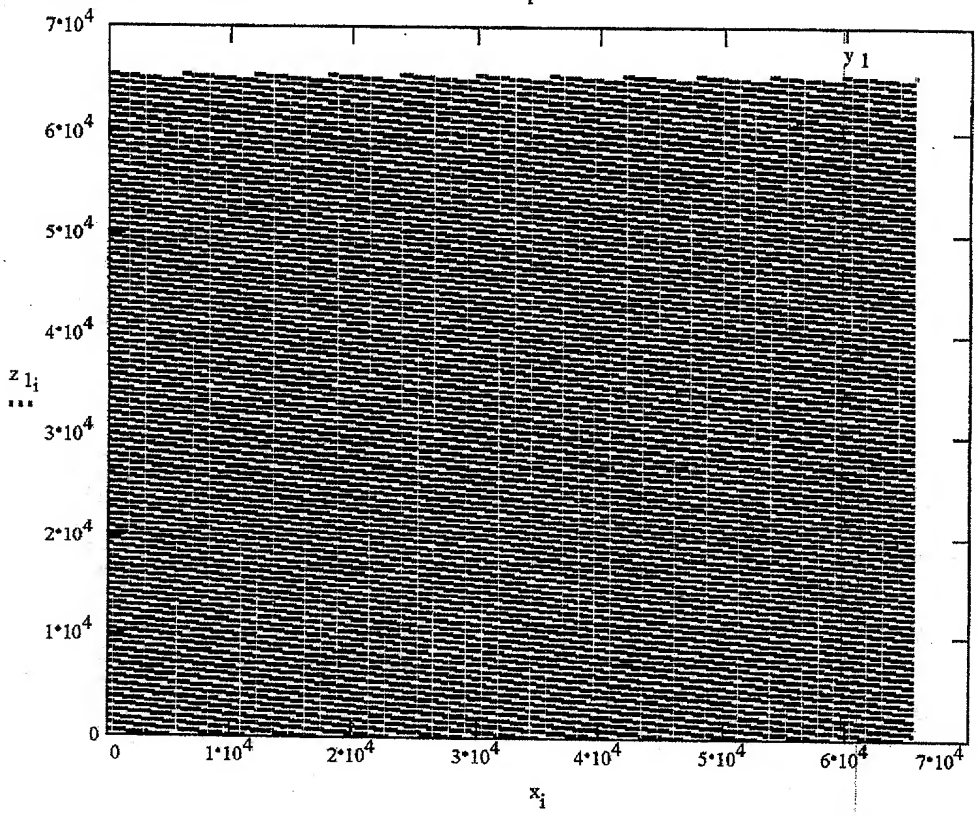
Linear Exception Region

J-13

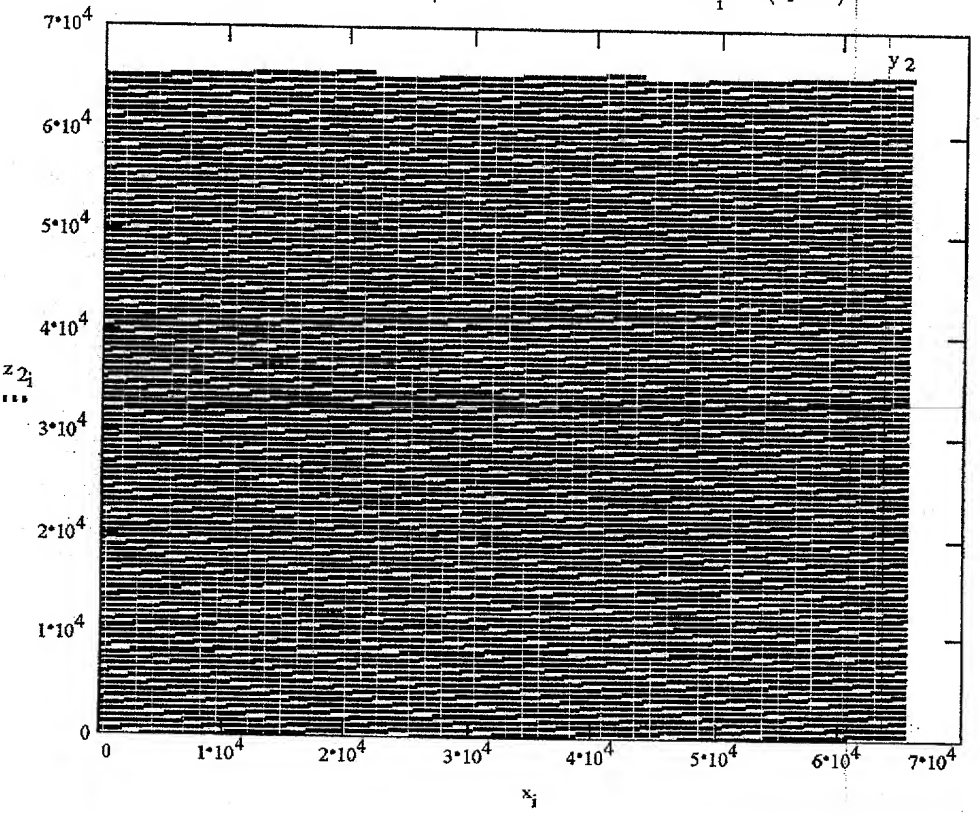
Multiplicative Group Operation (pseudogroup)
f(x,y) = xy mod 2^N-k, where x,y ^N-k;
= x where x < 2^N-k. (y always < 2^N-k.)

N = 16 bits $2^N = 65536$ k = 15 p = 65521 prime modulus

pseudogroup operation with key y1: $y_1 = 59624$ $z1_i := f(x_i, y_1)$

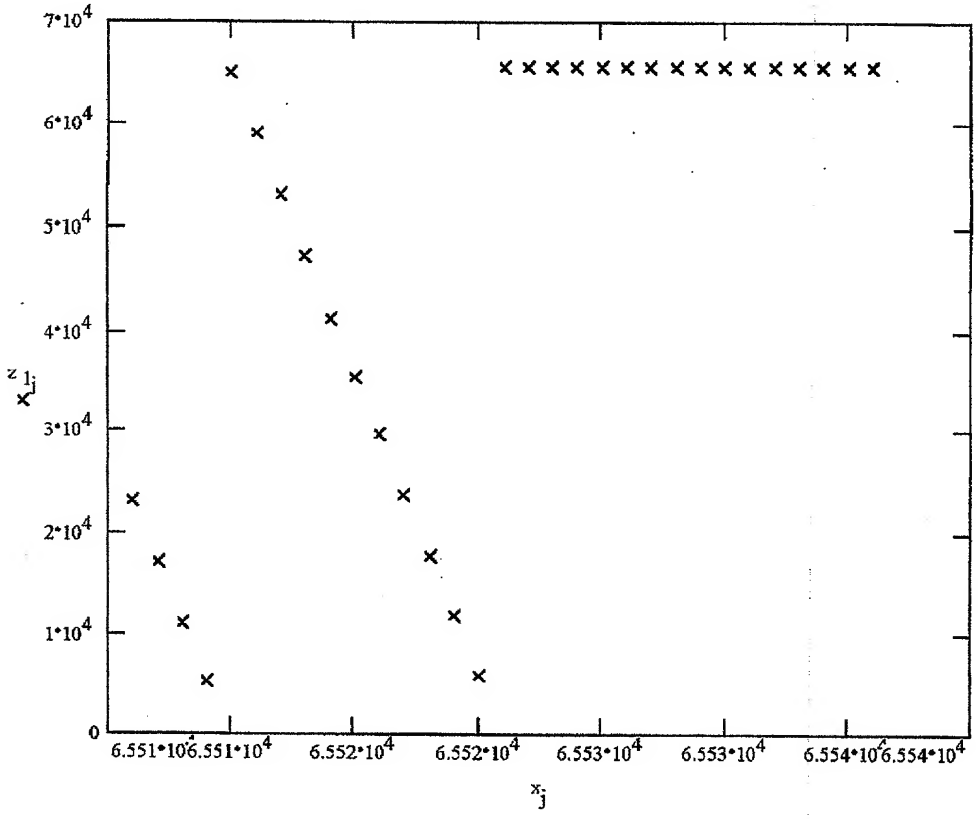


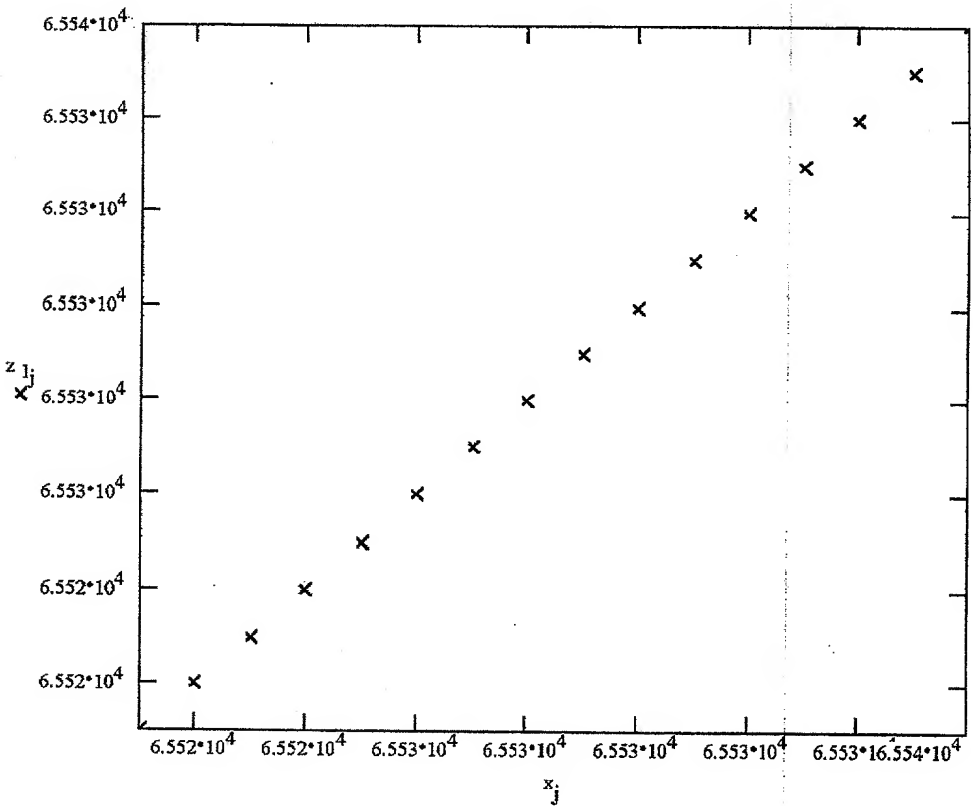
pseudogroup operation with key y2: $y_2 = 63631$ $z2_i := f(x_i, y_2)$



Zoom in on exception region of plot with key Y1

$j := 2^N - 2 \cdot k .. 2^N$



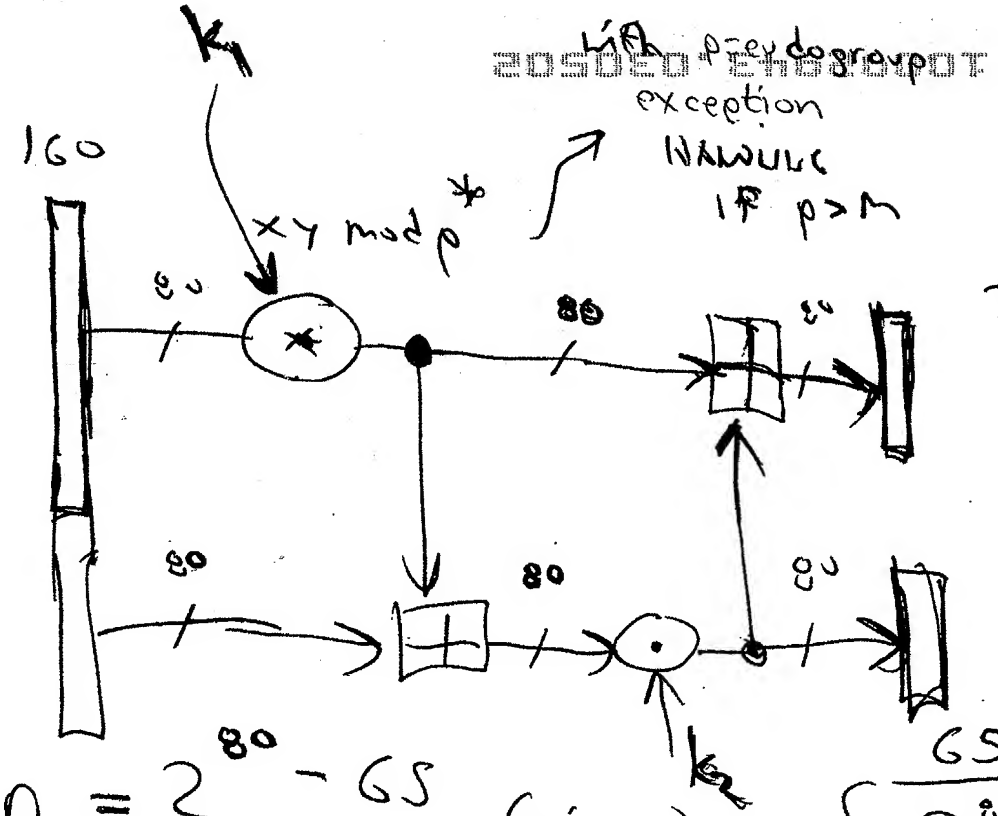


A post to sci.crypt suggested that increasing the IDEA subblock size to 32-bit subblocks from the design of 16 bits would increase the security of the IDEA algorithm to a factor of 2^{32}. Lai answered that the strength of the algorithm was based on the fact that $2^{16} + 1$ is a prime, whereas $2^{32}+1$ is not. Lai suggests that the stronger properties of the algorithm would be compromised. The point is that small changes in structure can have adverse ripple effects on the cryptographic structure that can become serious implementation errors. We look at other implementation errors in Chapter 13.

Not so limited with pseudogroup

$xy \bmod 2^N \pm k$

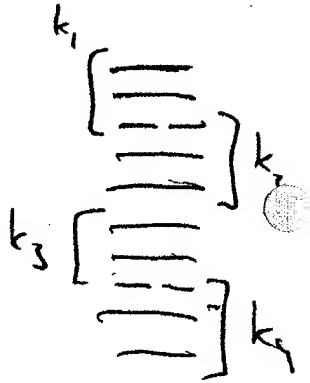
modification to ⊙



$p = 2^{80} - 65$ (prime)

$\frac{65}{2^{80}}$ = 1 of every 1.9×10^{22} keys is invalid

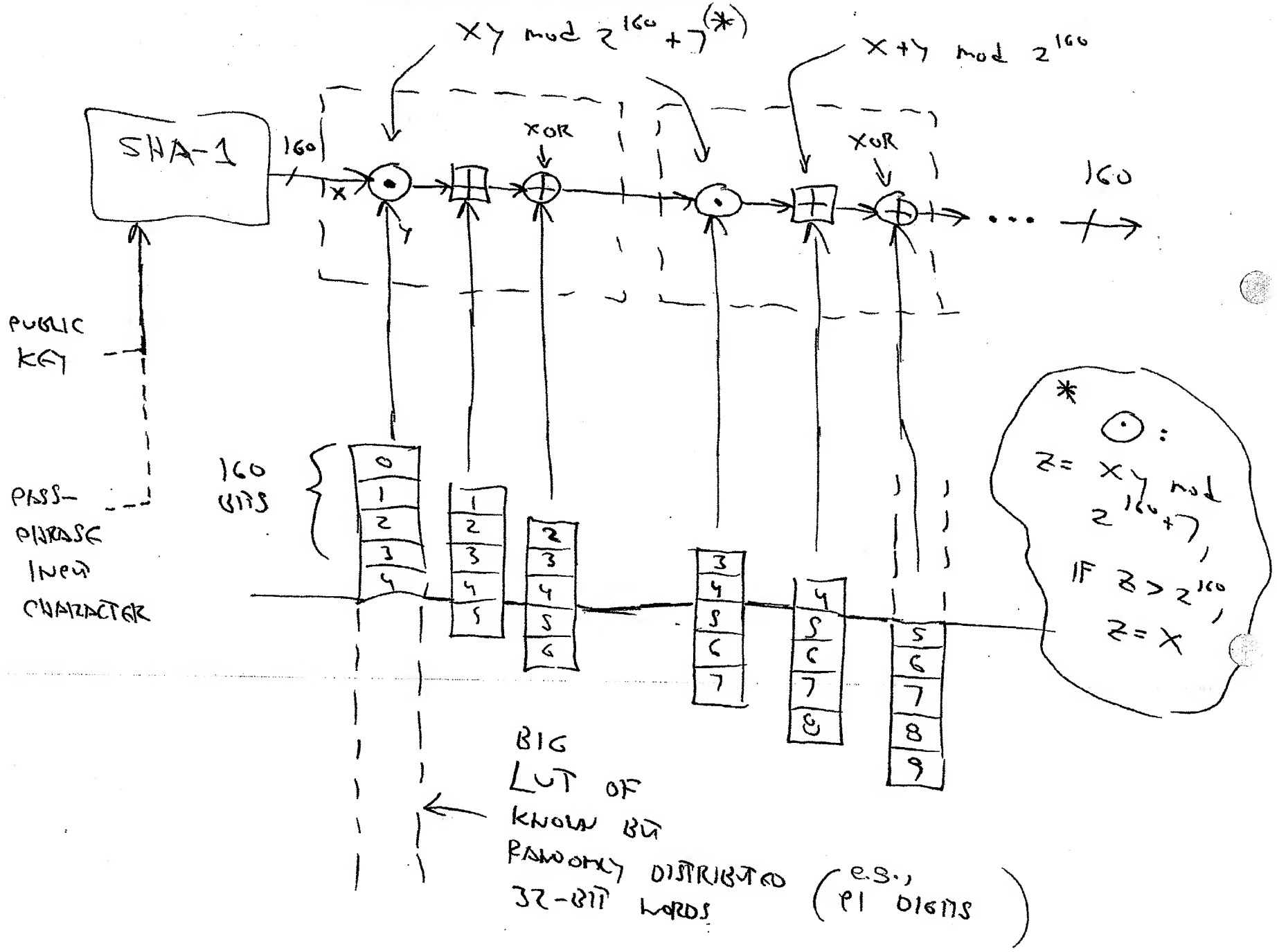
Just set MSB = 0 if invalid

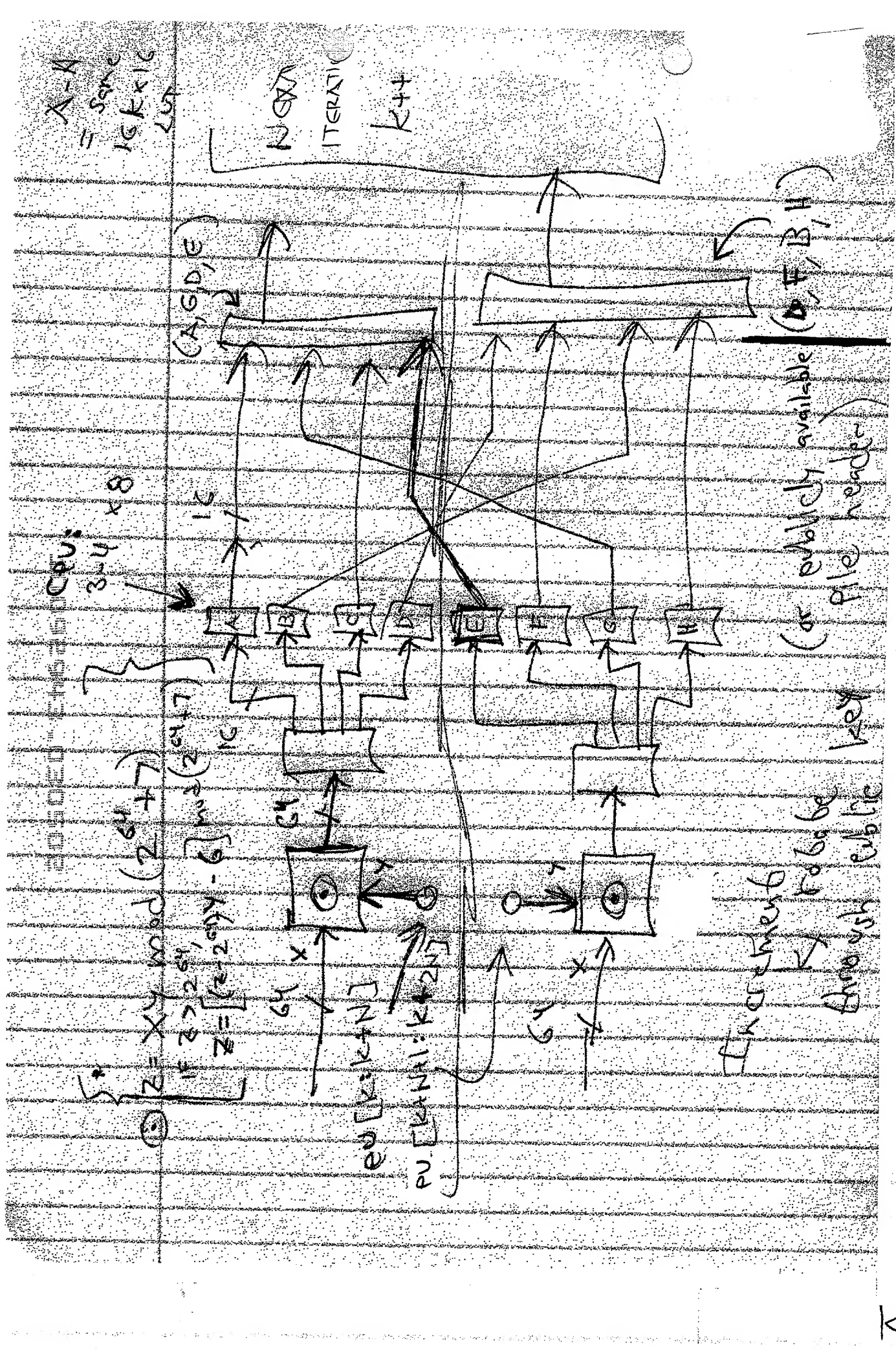


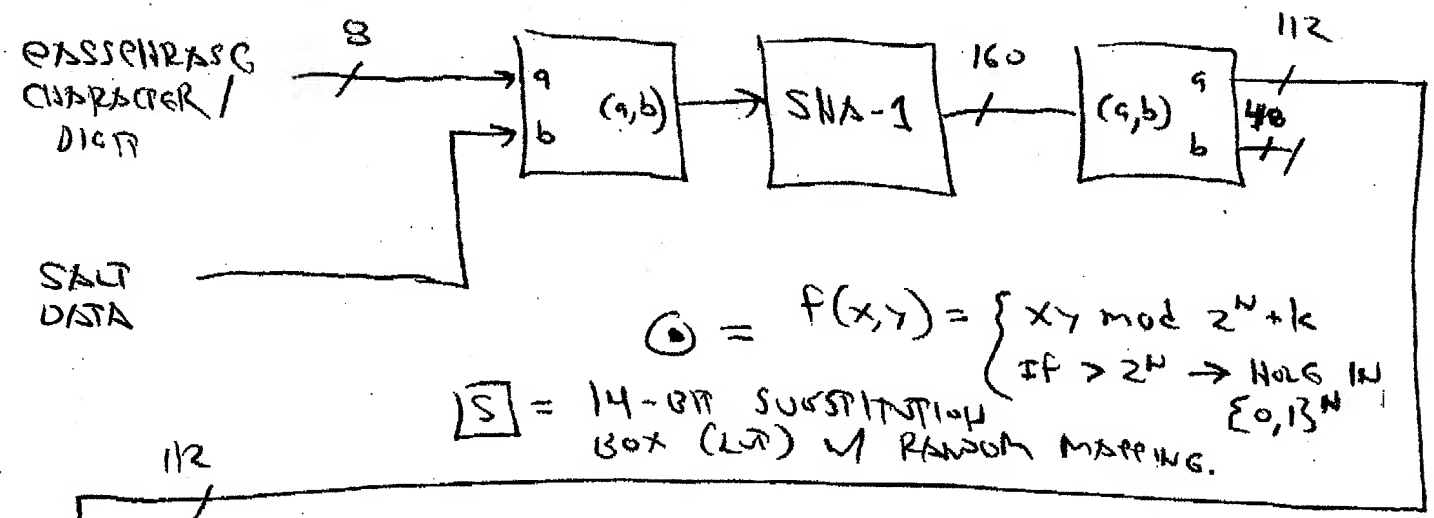
$(p - 64 < key \leq p)$

* [$p = 2^{80} + 13$ (prime) ← will need rarely / compute holes for key

K-1


XY mod $2^{160}+7$ (*)
X+Y mod 2^{160}
SHA-1
160
X
XOR
XOR
160
PUBLIC KEY
PASS-PHRASE INPUT CHARACTER
160 BITS
0 1 2 3 4
1 2 3 4 5
2 3 4 5 6
3 4 5 6 7
4 5 6 7 8
5 6 7 8 9
BIG LUT OF KNOWN BUT RANDOMLY DISTRIBUTED 32-BIT WORDS (e.g., PI DIGITS)
* ⊙ : Z = XY mod $2^{160}+7$, IF Z > 2^{160}, Z = X



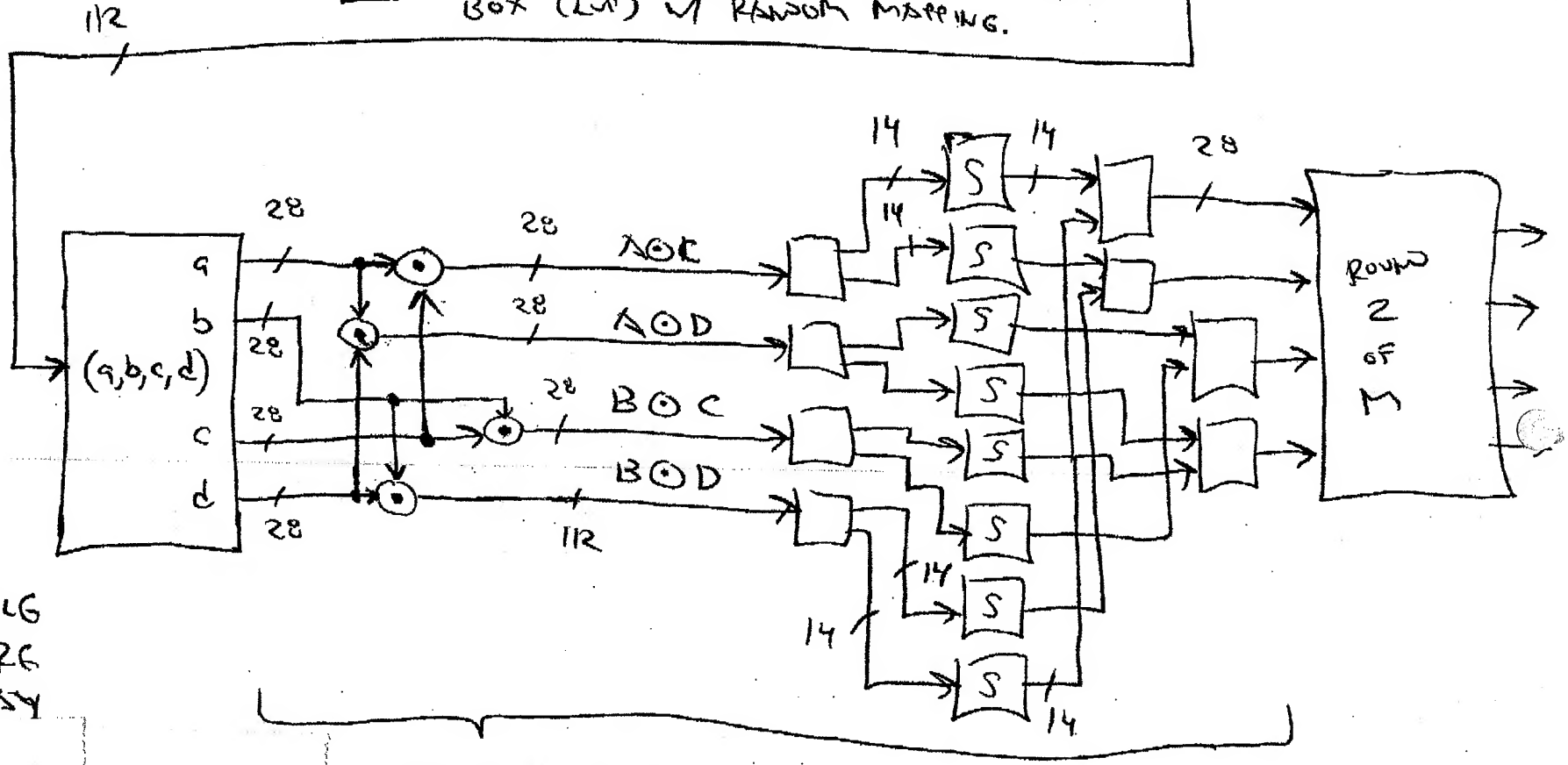


$(a+b)\cdot(c+d)$

$= ac + ad + bc + bd$

$\odot = f(x,y) = \begin{cases} xy \bmod 2^N + k \\ \text{if} > 2^N \rightarrow \text{HOLD IN } [0,13^N] \end{cases}$

$[S]$ = 14-BIT SUBSTITUTION BOX (LUT) W/ RANDOM MAPPING.



SIMPLE SECURE DELAY

ROUND 1 OF M

16K x 16, all [S] same (?)

Bit Length

Test Number (Y=prime, N=not prime, blank=not tested)

N	2^N+1	2^N+3	2^N+5	2^N+7	2^N+9	2^N+11	2^N+13	2^N+15	2^N+51
16	Y	Y							
17	N								
18	N	Y							
19	N								
20	N	N	N	Y					
21	N								
22	N								
23	N								
24	N	N	N	N	N	N	N	N	
25	N								
26	N								
27	N								
28	N	Y							
29	N								
30	N								
31	N								
32	N	N	N	N	N	N	N	Y	
40	N								
48	N	N	N	N	N	N	N	N	
56	N	N	N	N	N	N	N	N	
64	N	N	N	N	N	N	Y		
96	N								
128	N	N	N	N	N	N	N	N	(+51)

Likelihood of Exception (-bits)

2^N	1	3	5	7	9	11	13	15	51
65536	15.0	14.0							
131072									
262144		16.0							
524288									
1048576				17.0					
2097152									
4194304									
8388608									
16777216									
33554432									
67108864									
134217728									
268435456		26.0							
536870912									
1073741824									
2147483648									
4294967296								28.0	
1.09951E+12									
7.20576E+16									
1.84467E+19									
7.92282E+28							60.2		
3.40282E+38									
#REF!									122.3
1									
1									
1									
1									
1									
1									
1									
1									
1									

Bit Length	Test Number (Y=prime, N=not prime, blank=not tested)								
N	**2^N-1**	**2^N-3**	**2^N-5**	**2^N-7**	**2^N-9**	**2^N-11**	**2^N-13**	**2^N-15**	**2^N-17**
16	N	N	N	N	N	N	N	Y	
17	Y								
18	N	N							
19	Y								
20	N	Y							
21	N	N							
22	N	Y							
23	N	N							
24	N	Y							
25	N	N							
26	N	N							
27	N	N							
28	N	N							
29	N	**Y**							
30	N	N							
31	Y								
32	N	N	**Y**						
33	N	N	N	N	N				
34	N	N	N	N	N				
35	N	N	N	N	N				
36	N	N	**Y**						
37	N	N	N	N	N				
38	N	N	N	N	N				
39	N	N	N	Y					
40	N	N	N	N	N	N	N	N	N
41	N	N	N	N	N				
42	N	N	N	N	N				
43	N	N	N	N	N				
44	N	N	N	N	N				
45	N	N	N	N	N				
46	N	N	N	N	N				
47	N	N	N	N	N				
48	N	N	N	N	N	N	N	N	N

Effective bit length for one "leaked" bit, per offset below 2^N								
2^N	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**
65536								12.0	
131072	16.0								
262144									
524288	18.0								
1048576		18.0							
2097152									
4194304		20.0							
8388608									
16777216		22.0							
33554432									
67108864									
134217728									
268435456									
536870912		**27.0**							
1073741824									
2147483648	**30.0**								
4294967296			**29.4**						
8589934592									
17179869184									
34359738368									
68719476736			**33.4**						
1.37439E+11									
2.74878E+11									
5.49756E+11				36.0					
1.09951E+12									
2.19902E+12									
4.39805E+12									
8.79609E+12									
1.75922E+13									
3.51844E+13									
7.03687E+13									
1.40737E+14									
2.81475E+14									

TABLE I-1: "Pseudogroup" Operation: p=11 (prime), m=3 bits

— KEY VALUES →

INPUT VALUES ↓

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	1	3	5
3	3	6	9	1	4	7	10	2
4	4	8	1	5	9	2	6	10
5	5	10	4	9	3	8	2	7
6	6	1	7	2	8	3	9	4
7	7	3	10	6	2	9	5	1
8	8	5	2	10	7	4	1	9

TABLE I-2: Key values AND holes associated with them

Each row contains input values producing a given output, except for holes (black squares).
Each column contains key values producing outputs for a given input, except for holes.

— KEY VALUES →

Output VALUES ↓

	1	2	3	4	5	6	7	8
1	1	6	4	3	■	2	8	7
2	2	1	8	6	7	4	5	3
3	3	7	1	■	5	6	2	■
4	4	2	5	1	3	8	■	6
5	5	8	■	4	1	■	7	2
6	6	3	2	7	■	1	4	■
7	7	■	6	■	8	3	1	5
8	8	4	■	2	6	5	■	1

A "hole" is an output value that will not occur for any in the set {1,2,...2^m} of possible key values, given a particular input value in that set.

Note from table I-1 and I-2 that the output value of 7 does not occur with key value of 2.

TABLE II-1: (PRIOR ART) **Multiplicative Group Operation x*y mod p, p=17 (prime), m=4 bits**

— KEY VALUES →

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	*16*
2	2	4	6	8	10	12	14	16	1	3	5	7	9	11	13	15
3	3	6	9	12	15	1	4	7	10	13	16	2	5	8	11	14
4	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
5	5	10	15	3	8	13	1	6	11	16	4	9	14	2	7	12
6	6	12	1	7	13	2	8	14	3	9	15	4	10	16	5	11
7	7	14	4	11	1	8	15	5	12	2	9	16	6	13	3	10
8	8	16	7	15	6	14	5	13	4	12	3	11	2	10	1	9
9	9	1	10	2	11	3	12	4	13	5	14	6	15	7	16	8
10	10	3	13	6	16	9	2	12	5	15	8	1	11	4	14	7
11	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
12	12	7	2	14	9	4	16	11	6	1	13	8	3	15	10	5
13	13	9	5	1	14	10	6	2	15	11	7	3	16	12	8	4
14	14	11	8	5	2	16	13	10	7	4	1	15	12	9	6	3
15	15	13	11	9	7	5	3	1	16	14	12	10	8	6	4	2
16	*16*	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

↓ INCV VALUES ↓

Unmodified Modulo

TABLE II-2: Product **Operation: p=19 (prime), m=4 bits**

KEY VALUES →

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	*16*
2	2	4	6	8	10	12	14	*16*	*18*	1	3	5	7	9	11	13
3	3	6	9	12	15	*18*	2	5	8	11	14	*17*	1	4	7	10
4	4	8	12	*16*	1	5	9	13	*17*	2	6	10	14	*18*	3	7
5	5	10	15	1	6	11	*16*	2	7	12	*17*	3	8	13	*18*	4
6	6	12	*18*	5	11	*17*	4	10	*16*	3	9	15	2	8	14	1
7	7	14	2	9	*16*	4	11	*18*	6	13	1	8	15	3	10	*17*
8	8	*16*	5	13	2	10	*18*	7	15	4	12	1	9	*17*	6	14
9	9	*18*	8	*17*	7	*16*	6	15	5	14	4	13	3	12	2	11
10	10	1	11	2	12	3	13	4	14	5	15	6	*16*	7	*17*	8
11	11	3	14	6	*17*	9	1	12	4	15	7	*18*	10	2	13	5
12	12	5	*17*	10	3	15	8	1	13	6	*18*	11	4	*16*	9	2
13	13	7	1	14	8	2	15	9	3	*16*	10	4	*17*	11	5	*18*
14	14	9	4	*18*	13	8	3	*17*	12	7	2	*16*	11	6	1	15
15	15	11	7	3	*18*	14	10	6	2	*17*	13	9	5	1	*16*	12
16	*16*	13	10	7	4	1	*17*	14	11	8	5	2	*18*	15	12	9

INPUT VALUES ↓

TABLE II-3: Key values and holes associated with them

Each row contains input values producing a given output, except for holes (black squares).
Each column contains key values producing outputs for a given input, except for holes.

KEY VALUES →

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	10	13	5	4	16	11	12	■	2	7	8	3	15	14	6
2	2	1	7	10	8	13	3	5	15	4	14	16	6	11	9	12
3	3	11	1	15	12	10	14	■	13	6	2	5	9	7	4	■
4	4	2	14	1	16	7	6	10	11	8	9	13	12	3	■	5
5	5	12	8	6	1	4	■	3	9	10	16	2	15	■	13	11
6	6	3	2	11	5	1	9	15	7	12	4	10	■	14	8	■
7	7	13	15	16	9	■	1	8	5	14	11	■	2	10	3	4
8	8	4	9	2	13	14	12	1	3	16	■	7	5	6	■	10
9	9	14	3	7	■	11	4	13	1	■	6	15	8	2	12	16
10	10	5	16	12	2	8	15	6	■	1	13	4	11	■	7	3
11	11	15	10	■	6	5	7	■	16	3	1	12	14	13	2	9
12	12	6	4	3	10	2	■	11	14	5	8	1	■	9	16	15
13	13	16	■	8	14	■	10	4	12	7	15	9	1	5	11	2
14	14	7	11	13	■	15	2	16	10	9	3	■	4	1	6	8
15	15	■	5	■	3	12	13	9	8	11	10	6	7	16	1	14
16	16	8	■	4	7	9	5	2	6	13	■	14	10	12	15	1

OUTPUT VALUES ↓

A "hole" is an output value that will not occur for any in the set {1,2,...2^m} of possible input values, given a particular Key value in that set.

input value

Unmodified Modulo

TABLE III-1: Product **Operation: p=37 (prime), m=5 bits**

— KEY VALUE →

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	*32*
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	*32*	*34*	*36*	1	3	5	7	9	11	13	15	17	19	21	23	25	27
3	3	6	9	12	15	18	21	24	27	30	*33*	*36*	2	5	8	11	14	17	20	23	26	29	*32*	*35*	1	4	7	10	13	16	19	22
4	4	8	12	16	20	24	28	*32*	*36*	3	7	11	15	19	23	27	31	*35*	2	6	10	14	18	22	26	30	*34*	1	5	9	13	17
5	5	10	15	20	25	30	*35*	3	8	13	18	23	28	*33*	1	6	11	16	21	26	31	*36*	4	9	14	19	24	29	*34*	2	7	12
6	6	12	18	24	30	*36*	5	11	17	23	29	*35*	4	10	16	22	28	*34*	3	9	15	21	27	*33*	2	8	14	20	26	*32*	1	7
7	7	14	21	28	*35*	5	12	19	26	*33*	3	10	17	24	31	1	8	15	22	29	*36*	6	13	20	27	*34*	4	11	18	25	*32*	2
8	8	16	24	*32*	3	11	19	27	*35*	6	14	22	30	1	9	17	25	*33*	4	12	20	28	*36*	7	15	23	31	2	10	18	26	*34*
9	9	18	27	*36*	8	17	26	*35*	7	16	25	*34*	6	15	24	*33*	5	14	23	*32*	4	13	22	31	3	12	21	30	2	11	20	29
10	10	20	30	3	13	23	*33*	6	16	26	*36*	9	19	29	2	12	22	*32*	5	15	25	*35*	8	18	28	1	11	21	31	4	14	24
11	11	22	*33*	7	18	29	3	14	25	*36*	10	21	*32*	6	17	28	2	13	24	*35*	9	20	31	5	16	27	1	12	23	*34*	8	19
12	12	24	*36*	11	23	*35*	10	22	*34*	9	21	*33*	8	20	*32*	7	19	31	6	18	30	5	17	29	4	16	28	3	15	27	2	14
13	13	26	2	15	28	4	17	30	6	19	*32*	8	21	*34*	10	23	*36*	12	25	1	14	27	3	16	29	5	18	31	7	20	*33*	9
14	14	28	5	19	*33*	10	24	1	15	29	6	20	*34*	11	25	2	16	30	7	21	*35*	12	26	3	17	31	8	22	*36*	13	27	4
15	15	30	8	23	1	16	31	9	24	2	17	*32*	10	25	3	18	*33*	11	26	4	19	*34*	12	27	5	20	*35*	13	28	6	21	*36*
16	16	*32*	11	27	6	22	1	17	*33*	12	28	7	23	2	18	*34*	13	29	8	24	3	19	*35*	14	30	9	25	4	20	*36*	15	31
17	17	*34*	14	31	11	28	8	25	5	22	2	19	*36*	16	*33*	13	30	10	27	7	24	4	21	1	18	*35*	15	*32*	12	29	9	26
18	18	*36*	17	*35*	16	*34*	15	*33*	14	*32*	13	31	12	30	11	29	10	28	9	27	8	26	7	25	6	24	5	23	4	22	3	21
19	19	1	20	2	21	3	22	4	23	5	24	6	25	7	26	8	27	9	28	10	29	11	30	12	31	13	*32*	14	*33*	15	*34*	16
20	20	3	23	6	26	9	29	12	*32*	15	*35*	18	1	21	4	24	7	27	10	30	13	*33*	16	*36*	19	2	22	5	25	8	28	11
21	21	5	26	10	31	15	*36*	20	4	25	9	30	14	*35*	19	3	24	8	29	13	*34*	18	2	23	7	28	12	*33*	17	1	22	6
22	22	7	29	14	*36*	21	6	28	13	*35*	20	5	27	12	*34*	19	4	26	11	*33*	18	3	25	10	*32*	17	2	24	9	31	16	1
23	23	9	*32*	18	4	27	13	*36*	22	8	31	17	3	26	12	*35*	21	7	30	16	2	25	11	*34*	20	6	29	15	1	24	10	*33*
24	24	11	*35*	22	9	*33*	20	7	31	18	5	29	16	3	27	14	1	25	12	*36*	23	10	*34*	21	8	*32*	19	6	30	17	4	28
25	25	13	1	26	14	2	27	15	3	28	16	4	29	17	5	30	18	6	31	19	7	*32*	20	8	*33*	21	9	*34*	22	10	*35*	23
26	26	15	4	30	19	8	*34*	23	12	1	27	16	5	31	20	9	*35*	24	13	2	28	17	6	*32*	21	10	*36*	25	14	3	29	18
27	27	17	7	*34*	24	14	4	31	21	11	1	28	18	8	*35*	25	15	5	*32*	22	12	2	29	19	9	*36*	26	16	6	*33*	23	13
28	28	19	10	1	29	20	11	2	30	21	12	3	31	22	13	4	*32*	23	14	5	*33*	24	15	6	*34*	25	16	7	*35*	26	17	8
29	29	21	13	5	*34*	26	18	10	2	31	23	15	7	*36*	28	20	12	4	*33*	25	17	9	1	30	22	14	6	*35*	27	19	11	3
30	30	23	16	9	2	*32*	25	18	11	4	*34*	27	20	13	6	*36*	29	22	15	8	1	31	24	17	10	3	*33*	26	19	12	5	*35*
31	31	25	19	13	7	1	*32*	26	20	14	8	2	*33*	27	21	15	9	3	*34*	28	22	16	10	4	*35*	29	23	17	11	5	*36*	30
32	*32*	27	22	17	12	7	2	*34*	29	24	19	14	9	4	*36*	31	26	21	16	11	6	1	*33*	28	23	18	13	8	3	*35*	30	25

INPUT VALUE ↓

Example of holes:
29, 31 do not occur given K=2

Overflowing values (>32), which would be remapped to holes with pseudogroup operation, are shown in italics.

2.1.1

TABLE III-2: Key values and holes associated with them

Each row contains keys values producing a given output, except for holes (black squares).
Each column contains key values producting outputs for a given input, except for holes.

KEY VALUES →

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	1	19	25	28	15	31	16	14		26	27		20	8	5	7	24		2	13	30	32	29	17	3	10	11	4	23	21	6	22
2	2	1	13	19	30	25	32	28	29	15	17	31	3	16	10	14	11		4	26	23	27	21		6	20	22	8	9	5	12	7
3	3	20	1	10	8	19	11	5	25	4	7	28	23	24	15	21		31	6	2	16	22	13	14	9	30		12	32	26	18	29
4	4	2	26	1	23	13	27	19	21	30		25	6	32	20	28	22	29	8	15	9	17	5	31	12	3	7	16	18	10	24	14
5	5	21	14	29	1	7	6		17	19	24	22	26	3	25		9	27	10	28	2	12		11	15	13	18	20	4	31	30	
6	6	3	2	20	16	1	22	10	13	8	14	19	9	11	30	5		25	12	4	32	7	26	28	18	23	29	24	27	15		21
7	7	22	27	11	31	32	1	24	9		4	16	29	19		12	20	23	14	17	25	2	18	8	21		3	28	13		5	6
8	8	4	15	2	9	26	17	1	5	23	31	13	12	27	3	19	7	21	16	30	18		10	25	24	6	14	32		20	11	28
9	9	23	3	30	24	20		15	1	12	21	10	32		8	26	31	19	18	6	11	29	2	5	27	16	25		22	4	17	13
10	10	5	28	21	2	14	12	29		1	11	7	15	6	13		18	17	20	19	4	24	31	22	30	26		3	8	25	23	
11	11	24	16	12	17	8	28	6	30	27	1	4		14	18	3	5	15	22	32		19	23	2			10	7	31	9	29	20
12	12	6	4	3	32	2	7	20	26	16	28	1	18	22	23	10	29	13	24	8	27	14	15	19		9	21	11	17	30		5
13	13	25	29	31	10		23		22	5	18		1	30	28	17	16	11	26	21	20	9	7		2	19	32	15	3	14	4	27
14	14	7	17	22	25	27	2	11	18	31	8	32	21	1		24	3	9	28		13	4		16	5	29	6	19	26		10	12
15	15	26	5	13	3	21	18	25	14	20		29	4	9	1	31	27	7	30	10	6		28		8	2	17	23	12	19	16	
16	16	8	30	4	18	15		2	10	9	25	26	24	17	6	1	14	5	32	23		31	20	13	11	12	28	27		3	22	19
17	17	27	18	32		9	13	16	6		15	23	7	25	11	8	1	3			29	26	12	30	14	22	2	31	21	24	28	4
18	18	9	6	23	11	3	29	30	2	24	5	20	27		16	15	25	1		12	22	21	4	10	17	32	13		7	8		26
19	19	28	31	14	26		8	7		13	32	17	10	4	21	22	12		1	25	15	16		27	20	5	24	2	30	29	3	11
20	20	10	19	5	4	28	24	21	31	2	22	14	30	12	26	29			3	1	8	11	25	7	23	15		6	16	13	9	
21	21	29	7		19	22	3		27	28	12	11	13	20	31		23	32	5	14	1	6	17	24	26	25	9	10	2		15	18
22	22	11	32	24		16	19	12	23	17	2	8		28		6	10	30	7	27	31	1	9	4	29		20	14	25	18	21	3
23	23	30	20	15	12	10		26	19	6	29	5	16		4	13		28	9	3	24		1	21	32	8	31	18	11	2	27	25
24	24	12	8	6	27	4	14	3	15	32	19	2		7	9	20	21	26	11	16	17	28	30	1		18	5	22		23		10
25	25	31			5		30	17	11	21	9		19	15	14	27	8	24	13	29	10	23	22	18	1	28	16	26	20	7	2	32
26	26	13	21	25	20	29	9	31	7	10			2	23	19		32	22	15	5	3	18	14		4	1	27	30	6	28	8	17
27	27	32	9	16		23	25	8	3		26	30	22	31	24	4	19	20	17	18		13	6	15	7	11	1		29	12	14	2
28	28	14		7	13	17	4	22		25	16	27	5	2	29	11	6	18	19	31	26	8		32	10	21	12	1	15		20	24
29	29		22		28	11	20		32	14	6	24	25	10		18	30	16	21	7	19	3	27	12	13	31	23	5	1	17	26	9
30	30	15	10	26	6	5		13	28	3		21	8	18	2	25	17	14	23	20	12		19	29	16	4		9	24	1	32	31
31	31			17	21		15	27	24	29	23	18	28	26	7	32	4	12	25		5	30	11	9	19	14	8	13	10	22	1	16
32	32	16	23	8		30	31	4	20	18	13	15	11		12	2	28	10	27	9		25	3	26	22	24	19	17		6	7	1

INPUT VALUES ↓

A "hole" is an output value that will not occur for any input value in the set {1,2,...2^m} of possible input values, given a particular Key value in that set.

Encryption (forward)

Sub-Block size ↓

a = plaintext x = key $M = 2^N$

1. $z = ax \bmod p$

p = prime just above M

2. If $z > M$,

2.1 $k = z - M$

2.2 $z = kx - (p - M - 1) \bmod p$

end.

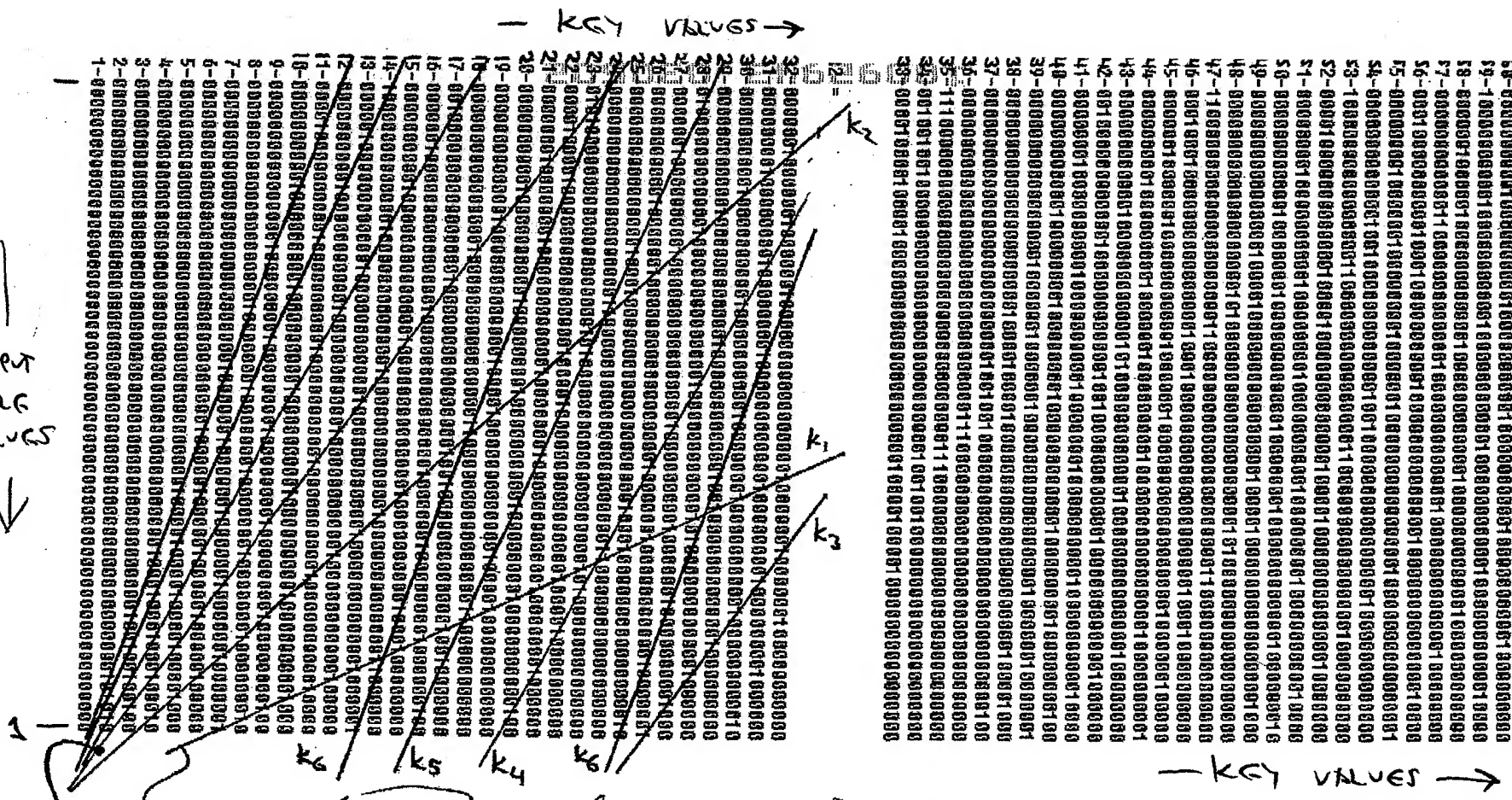
Decryption (inverse)

z = ciphertext x^{-1} = inverse key

1. $k = x^{-1}(z + (p - M - 1)) \bmod p$

2. If $1 \le k \le (p - M - 1)$, 2.1 $z = M + k$; end

2.2 $a = z x^{-1} \bmod p$



$p = 71$

$M = 64$

$(71 - 65 = 6)$ ← $2^N + 1$ is special case where $\Sigma k = 0$

Examples:

$k_1 = 1$

$k_2 = 2$

$\vdots$

$k_6 = 6$

$Y_1 = 1 \cdot (x-6) = 1 \cdot x - 6$

$Y_2 = 2 \cdot (x-3) = 2 \cdot x - 6$

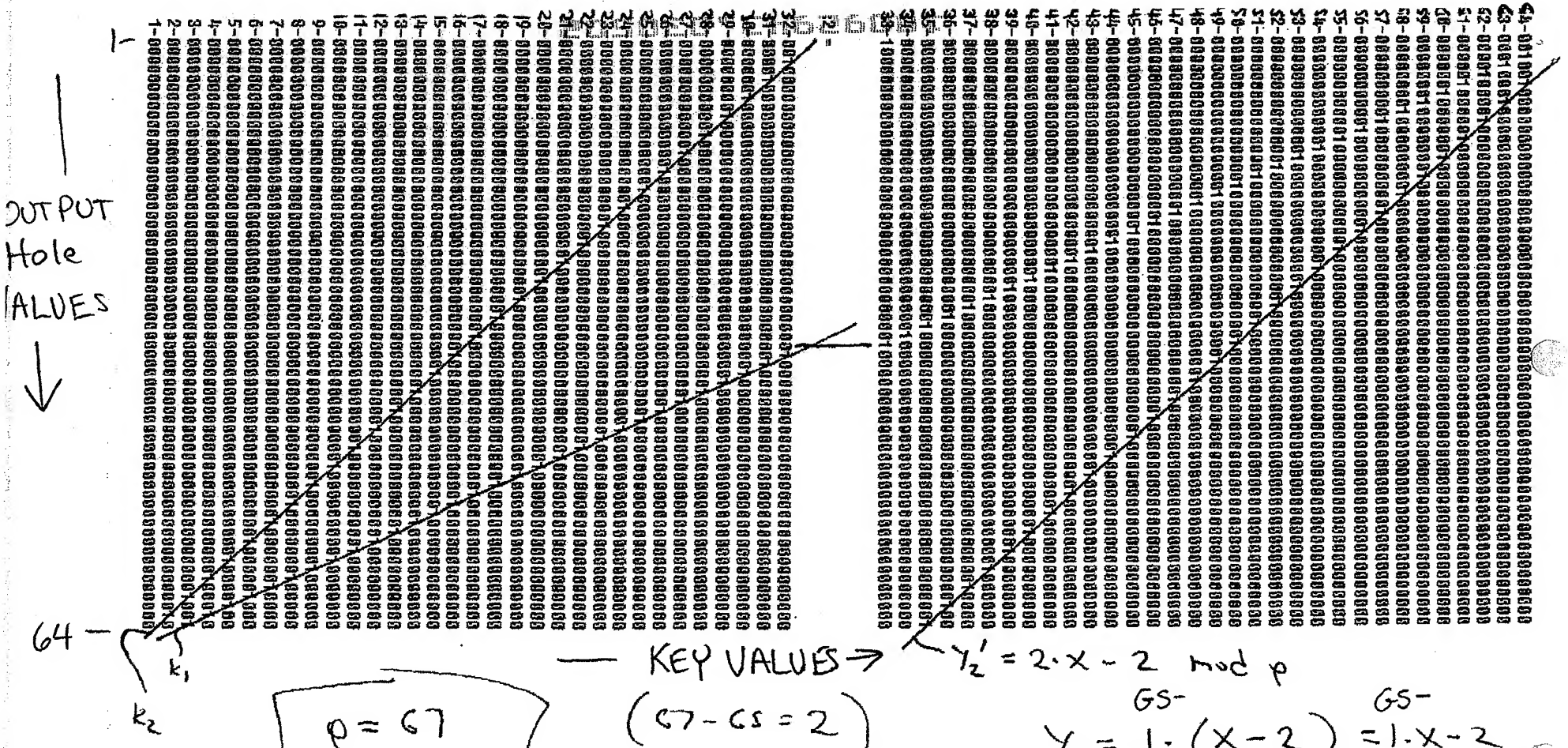
$Y_3 = 3 \cdot (x-2) = 3 \cdot x - 6$

$Y_4 = 4 \cdot x - 6$

$Y_5 = 5 \cdot x - 6$

$Y_6 = 6 \cdot x - 6$

$\therefore Y_k = k \cdot x - (p - (M+1))$



$p = 67$
$M = 64$

$(67 - 65 = 2)$

$k_1 = 1$
$k_2 = 2$

$Y_{21} = Y_{11} + k_1$

$Y_{22} = Y_{12} + k_2$

$Y_1 = 1 \cdot (X - 2) \overset{GS-}{=} 1 \cdot X - 2$

$Y_2 = 2 \cdot (X - 1) \overset{GS-}{=} 2 \cdot X - 2$

A PARTICULARLY USEFUL PSEUDOGROUP EQUATION:

$$Y(X,K) = M+1 - \left[KX - (p - (M+1)) \right] \mod p$$

IF $Y(X,K) > M$, NO HOLE

P-2

$y_1 = 1 \cdot x - 8$

$y_2 = 2 \cdot x - 8$

$y_3 = 3 \cdot x - 8$

$\therefore y_2 = 2 \cdot x - 8$

$y_8(x=3) = 16$

(3, 16) should be a hole - it is!

$y = 7 \cdot 51 - 8$ and 73

For $x = 51$, $k = 7$

$73 - 65 = 8$

$M = 64$

$Q = 73$

$= 65$ (in base)

$x = 52, k = 7$

$= 64$

```
function h = holes(y,p,M,short);

% h = holes(y,p,N, short);
% CONFIDENTIAL AND PROPRIETARY
% Edwin A. Suominen

% Finds "holes" - skipped values of set {0,1}^N in result
% of x*y mod p.

% Number of values in set S:{0,1}^N
% M = 2^N;

% For vector inputs...
for k=1:length(y)

s = 1:M;    % Working array of values in set S

% Zero out values in set that occur ("non-holes")
  for i = 1:M
    j = product(i,y(k),p);  % xy mod p
    % Zero out if not a hole
    if j<=M, s(j) = 0; end
  end

% Sort decending to get holes first
  z = -sort(-s);

if nargin>3,
 % There can be no more than p-2^N-1 holes.
 % Limit size of result vector(s) accordingly.
   z = z(1:p-M-1);
% Add result vector to array (if vector y)
  h(:,k) = z';
else
  h(:,k) = s';
end



end

end
```

```
function [y1,y2] = holeplot(p,M);

% y = holeplot(p,M)

% Modulus is 2^N+k, where k is odd
% p = M + k;

% Create 1010...matrix of holes
for i = 1:M/2,
  % Get holes for each column
  s = holes(M+1-i,p,M);
  % Convert to 1010... format
  s = s>0;
  % Convert to text string
  col = M+1-i
  if col<10,
    rl = [' ' num2str(col,2)];
  else
    rl = num2str(col,2);
  end
  y1(i,:) = [ rl '-' num2str(s,1)];
end

for i = 1:M/2,
  % Get holes for each column
  s = holes(M+1-(i+M/2),p,M);
  % Convert to 1010... format
  s = s>0;
  % Convert to text string
  col = M/2+1-i
  if col<10,
    rl = [' ' num2str(col,2)];
  else
    rl = num2str(col,2);
  end
  y2(i,:) = [ rl '-' num2str(s,1)];
end
```

HOLES.M

```
## usage: h = holes(y,p,M)
## Function discovered by Edwin A. Suominen
## Written for Octave (GNU MATLAB alternative)

function [h,yi] = holes(y,p,M)

h = zeros(p-M-1,2);

## Compute inverse of y mod p
[d,yi]= gcd(p,y);
if ( yi(2) < 0 )
  yi = p + yi(2);
elseif
  yi = yi(2);
endif

## Compute column 1 of LUT for this key y:
## holes in ascending order
kk = 0; # Counter for iterating next valid hole value
## For all possible hole values...
for i = 1:p-M-1
  ## Compute prospective hole value (may not be valid)
  h(i,1) = M+1 - rem( i*y-(p-M-1) ,p);
  ## If not valid (if >M), set to flag value
  if ( (h(i,1)>M) | (h(i,1)<1) )
    h(i,:) = zeros(1,2);
  else
    kk++; # Increment valid holes counter
  endif
endfor

## Compute column 2 of LUT for this key y: all possible
## overflowing values, xy mod p > M, in ascending order
kk2 = 1; # Counter for iterating next poss. overflow value
## For M+1 (lowest overflow) to p-1 (highest possible)...
for i = M+1:p-1
  ## Compute input value that would produce each
  ## possible overflowing output
  if ( rem(i*yi,p) <= M ) # If input valid...
    h(kk2,2) = i; # ...assign overflow to LUT.
    kk2++; # Move to next available LUT entry
  endif
endfor

## Sort ascending by values in each column
h = sort(h);
if ( 1+(length(h)-kk) > length(h) )
```

```
  h = 0; # If there are no holes (e.g., for y=1)
else
  h = h(1+(length(h)-kk):length(h),:); # Shrink h to omit flag values
endif

endfunction
```

ENCRYPT.M

```
## usage: z = encrypt (x,y,N,k)
## INPUT: input block(s) x, key y, block length N in bits,
## k offset of modulus from 2^N (p=2^N+k)
## OUTPUT: z = x*y mod (2^N+k), but if
## z >= p, z = ( (z-2^N)*y - (k-1) ) mod (2^N+k )
## Function discovered by Edwin A. Suominen
## Written for Octave (GNU MATLAB alternative)

function z = encrypt(x,y,N,k)

L = length(x); # multiple input blocks can be supplied in a vector
z = zeros(L,1); # initialize output vector

## Enforce k must be odd
if ((k/2)==floor(k/2))
  disp('2^N+k cannot be prime is k is even!');
  return;
endif

## Define set order (M) and modulus (p)
M = 2^N; p = M+k;

## Compute LUT of holes in ascending order
## for this key y
h = holes(y,p,M);
Nh = rows(h);

## Basic modulo multipication operation
## Do as array to speed things up
z = rem(x.*y,p);

## For each element in vector...
for i = 1:L

  ## Inventive exception handling
  if (z(i) > M)
    ## Map overflowing value to corresponding hole value in LUT
    ## If there are no holes (h=1 scalar), this code will not
    ## be called because z will always be <= M.
    c = 1:Nh; c = c'; # 1,2 ... (# of valid holes)
    c = c .* ( (z(i)*ones(Nh,1))==h(:,2) ); # Zeros with index of match
    ## z = hole from LUT entry having matching overflow value
    z(i) = h(max(c),1);
  endif

endfor
endfunction
```

DECRYPT.M

```
## usage: x = decrypt (z,y,N,k)
## INPUT: encoded block(s) z, key y, block length N in bits,
## k offset of modulus from 2^N (p=2^N+k)
## OUTPUT: x = z*y^-1 mod (2^N+k), but if
## z = h, where h = ( ((1:k)*y - (k-1) ) mod (2^N+k )
## then z = M+a, where
## a = y^-1 * (2*M+(2+p-h)) mod (2^N+k)
## Function discovered by Edwin A. Suominen
## Written for Octave (GNU MATLAB alternative)

function x = decrypt (z,y,N,k)

L = length(z);  # multiple input blocks can be supplied in a vector

## Enforce k must be odd
if ((k/2)==floor(k/2))
  disp('2^N+k cannot be prime is k is even!');
  return;
endif

## Define set order (M) and modulus (p)
M = 2^N; p = M+k;

## Compute LUT of holes in ascending order
## for this key y
[h,y] = holes(y,p,M); # With two args out, returns y^-1
if ( size(h)==1 )
  Nh = 0; # Account for special case of no holes
else
  Nh = rows(h);
endif

## Done with encryption key y, now y is modulo inverse of orig. y

## For all encrypted blocks (values)...
for i = 1:L

  if Nh>0
    ## If z(i) has been mapped to a hole, restore to overflowing value
    ## For all possible hole values given this key
    for j = 1:Nh
      ## If matches a hole value, remap back
      if (z(i)==h(j,1)), z(i) = h(j,2); endif
    endfor
  endif

endfor
```

```
## Now invert remapped values in vector
## Restored overflowing values will be decrypted properly.
## Do as array to speed things up
x = rem(z.*y,p);      # y = y^1 at this point

endfunction
```

HOLETEST.M

```
## HOLETEST.M
## Written for Octave (GNU MATLAB alternative)

# Np = 1; M = 128; p = M + 3;
Np = 1; M = 512; p = M + 9;

% Try all column (key) values in {1,2,...M}
for j = 2:M,

% Get hole values with brute-force lookup method
x1 = holes1(j,p,M);

% Get hole values using formula discovered by Ed Suominen
x2 = holes2(j,p,M);

disp('');
disp(['j=',num2str(j)]);
disp('  -x1-  -x2-');
disp([x1 x2]);

% Compare
err(j) = sum(abs(x1-x2));
disp(['Sum of absolute differences = ',num2str(err(j))]);

endfor
```

HOLES1.M

```
function h = holes1(y,p,M);

% h = holes1(y,p,N);
% Finds "holes" - skipped values of set {0,1}^N in result
% of x*y mod p. Variable length result with only holes.

% Number of values in set S:{0,1}^N
% M = 2^N;

s = 1:M;    % Working array of values in set S

% Zero out values in set that occur ("non-holes")
  for i = 1:M
    j = rem(i*y,p);  % xy mod p
    % Zero out if not a hole
    if j<=M, s(j) = 0; end
  endfor

% Sort decending to get holes first
h = -sort(-s);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz)';

endfunction
```

HOLES2.M

```
function h = holes2(y,p,M);

% h = holes2(y,p,M)
% Finds "holes" - skipped values of set {0,1}^N in result
% of x*y mod p.
% Uses equation discovered by Edwin A. Suominen

% Number of values in set S:{0,1}^N
% M = 2^N;

k = p-(M+1);

% For vector inputs...
for i=1:length(y)

  for j=1:k,

    ## Input values between M+1 and p will of necessity
    ## be mapped to holes (values not produced by inputs
    ## from set {1,2,...M} because xy mod p is a bijection
    ## (See HAC 1.8 Definition)
    ## h(j,i) = rem( (M+j)*y ,p);
    ## Equation above is simple but doesn't work when
    ## M < xy < p (which happens rarely, but it happens).

    h(j,i) = M+1 - rem(j*y(i)-k,p);
  endfor

endfor

% Map negs. to 0, Sort decending to match formats
Nok = sum(h<=M); h = sort(h); h = h(1:Nok);
if Nok==0, h = []; endif
h = h.*(h>0);
h = -sort(-h);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz);

endfunction
```

1093-093000-1.TXT

```
octave:56> date
ans = [redacted]
octave:57> clock
ans =

  2000.0000           9.0000          30.0000
12.0000    35.0000    32.0890

octave:58> type holes1
holes1 is the user-defined function
defined from: /1093-2/holes1.m

function h = holes1(y,p,M);

% h = holes1(y,p,N);
% CONFIDENTIAL AND PROPRIETARY
% Edwin A. Suominen
% 091600 - Initial writing
% Finds "holes" - skipped values of set
{0,1}^N in result
% of x*y mod p. Variable length result
with only holes.

% Number of values in set S:{0,1}^N
% M = 2^N;

s = 1:M;    % Working array of values in
set S

% Zero out values in set that occur
("non-holes")
  for i = 1:M
    j = rem(i*y,p);  % xy mod p
    % Zero out if not a hole
    if j<=M, s(j) = 0; end
  endfor


% Sort decending to get holes first
h = -sort(-s);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz)';

endfunctionoctave:59> type holes2
holes2 is the user-defined function
defined from: /1093-2/holes2.m

function h = holes2(y,p,M);

% h = holes2(y,p,M)
[redacted]
```

```
% Finds "holes" - skipped values of set
{0,1}^N in result
% of x*y mod p.
% Uses equation discovered by EAS [redacted]

% Number of values in set S:{0,1}^N
% M = 2^N;

k = p-(M+1);

% For vector inputs...
for i=1:length(y)

  for j=1:k,

    ## Input values between M+1 and p
will of necessity
    ## be mapped to holes (values not
produced by inputs
    ## from set {1,2,...M} because xy mod
p is a bijection
    ## (See HAC 1.8 Definition)
    h(j,i) = rem( (M+j)*y ,p);

    ## The simple equation above is
substituted for the one below
    ## h(j,i) = M+1 - rem(j*y(i)-k,p);
  endfor

endfor

% Map negs. to 0, Sort decending to match
formats
Nok = sum(h<=M); h = sort(h); h =
h(1:Nok);
if Nok==0, h = []; endif
h = h.*(h>0);
h = -sort(-h);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz);

endfunctionoctave:60> type holetest
holetest is the script file: /1093-
2/holetest.m

## HOLETEST.M
##   This file is CONFIDENTIAL AND
PROPRIETARY.
[redacted]
##   Written for Octave (GNU MATLAB
alternative)
## REVISION [redacted]
```

```
Np = 1; M = 512; p = M + 9;

% Try all column (key) values in
{1,2,...M}
for j = 2:M,

% Get hole values with brute-force lookup
method
x1 = holes1(j,p,M);

% Get hole values using formula
discovered [redacted]
% by Ed Suominen
x2 = holes2(j,p,M);

disp('');
disp(['j=',num2str(j)]);
disp('  -x1-  -x2-');
disp([x1 x2]);


% Compare
err(j) = sum(abs(x1-x2));
disp(['Sum of absolute differences =
',num2str(err(j))]);

endforoctave:61> who

*** currently compiled functions:

clock   date    holes1  holes2

octave:62> holetest

j=2
  -x1-  -x2-
  511  511
  509  509
  507  507
  505  505
Sum of absolute differences = 0

j=3
  -x1-  -x2-
  512  512
  509  509
  506  506
  503  503
  500  500
  497  497
Sum of absolute differences = 0

j=4
  -x1-  -x2-
  509  509
  505  505
  501  501
  497  497
  493  493
  489  489
Sum of absolute differences = 0

j=5
  -x1-  -x2-
  511  511
  506  506
  501  501
  496  496
  491  491
  486  486
  481  481
Sum of absolute differences = 0

j=6
  -x1-  -x2-
  509  509
  503  503
  497  497
  491  491
  485  485
  479  479
  473  473
Sum of absolute differences = 0

j=7
  -x1-  -x2-
  507  507
  500  500
  493  493
  486  486
  479  479
  472  472
  465  465
Sum of absolute differences = 0

j=8
  -x1-  -x2-
  505  505
  497  497
  489  489
  481  481
  473  473
  465  465
  457  457
Sum of absolute differences = 0

j=9
  -x1-  -x2-
  512  512
  503  503
  494  494
  485  485
  476  476
```

```
  467  467
  458  458
  449  449
Sum of absolute differences = 0

j=10
  -x1-  -x2-
  511  511
  501  501
  491  491
  481  481
  471  471
  461  461
  451  451
  441  441
Sum of absolute differences = 0

j=11
  -x1-  -x2-
  510  510
  499  499
  488  488
  477  477
  466  466
  455  455
  444  444
  433  433
Sum of absolute differences = 0

j=12
  -x1-  -x2-
  509  509
  497  497
  485  485
  473  473
  461  461
  449  449
  437  437
  425  425
Sum of absolute differences = 0

j=13
  -x1-  -x2-
  508  508
  495  495
  482  482
  469  469
  456  456
  443  443
  430  430
  417  417
Sum of absolute differences = 0

j=14
  -x1-  -x2-
  507  507
  493  493
  479  479
  465  465
  451  451
  437  437
  423  423
  409  409
Sum of absolute differences = 0

j=15
  -x1-  -x2-
  506  506
  491  491
  476  476
  461  461
  446  446
  431  431
  416  416
  401  401
Sum of absolute differences = 0

j=16
  -x1-  -x2-
  505  505
  489  489
  473  473
  457  457
  441  441
  425  425
  409  409
  393  393
Sum of absolute differences = 0

j=17
  -x1-  -x2-
  504  504
  487  487
  470  470
  453  453
  436  436
  419  419
  402  402
  385  385
Sum of absolute differences = 0

j=18
  -x1-  -x2-
  503  503
  485  485
  467  467
  449  449
  431  431
  413  413
  395  395
  377  377
Sum of absolute differences = 0

j=19
  -x1-  -x2-
  502  502
```

```
  162  162
  108  108
   54   54
Sum of absolute differences = 0

j=468
  -x1-  -x2-
  424  424
  371  371
  318  318
  265  265
  212  212
  159  159
  106  106
   53   53
Sum of absolute differences = 0

j=469
  -x1-  -x2-
  416  416
  364  364
  312  312
  260  260
  208  208
  156  156
  104  104
   52   52
Sum of absolute differences = 0

j=470
  -x1-  -x2-
  408  408
  357  357
  306  306
  255  255
  204  204
  153  153
  102  102
   51   51
Sum of absolute differences = 0

j=471
  -x1-  -x2-
  400  400
  350  350
  300  300
  250  250
  200  200
  150  150
  100  100
   50   50
Sum of absolute differences = 0

j=472
  -x1-  -x2-
  392  392
  343  343
  294  294
  245  245
  196  196
  147  147
   98   98
   49   49
Sum of absolute differences = 0

j=473
  -x1-  -x2-
  384  384
  336  336
  288  288
  240  240
  192  192
  144  144
   96   96
   48   48
Sum of absolute differences = 0

j=474
  -x1-  -x2-
  376  376
  329  329
  282  282
  235  235
  188  188
  141  141
   94   94
   47   47
Sum of absolute differences = 0

j=475
  -x1-  -x2-
  368  368
  322  322
  276  276
  230  230
  184  184
  138  138
   92   92
   46   46
Sum of absolute differences = 0

j=476
  -x1-  -x2-
  360  360
  315  315
  270  270
  225  225
  180  180
  135  135
   90   90
   45   45
Sum of absolute differences = 0

j=477
  -x1-  -x2-
  352  352
```

```
  308  308
  264  264
  220  220
  176  176
  132  132
   88   88
   44   44
Sum of absolute differences = 0

j=478
  -x1-  -x2-
  344  344
  301  301
  258  258
  215  215
  172  172
  129  129
   86   86
   43   43
Sum of absolute differences = 0

j=479
  -x1-  -x2-
  336  336
  294  294
  252  252
  210  210
  168  168
  126  126
   84   84
   42   42
Sum of absolute differences = 0

j=480
  -x1-  -x2-
  328  328
  287  287
  246  246
  205  205
  164  164
  123  123
   82   82
   41   41
Sum of absolute differences = 0

j=481
  -x1-  -x2-
  320  320
  280  280
  240  240
  200  200
  160  160
  120  120
   80   80
   40   40
Sum of absolute differences = 0

j=482
  -x1-  -x2-
  312  312
  273  273
  234  234
  195  195
  156  156
  117  117
   78   78
   39   39
Sum of absolute differences = 0

j=483
  -x1-  -x2-
  304  304
  266  266
  228  228
  190  190
  152  152
  114  114
   76   76
   38   38
Sum of absolute differences = 0

j=484
  -x1-  -x2-
  296  296
  259  259
  222  222
  185  185
  148  148
  111  111
   74   74
   37   37
Sum of absolute differences = 0

j=485
  -x1-  -x2-
  288  288
  252  252
  216  216
  180  180
  144  144
  108  108
   72   72
   36   36
Sum of absolute differences = 0

j=486
  -x1-  -x2-
  280  280
  245  245
  210  210
  175  175
  140  140
  105  105
   70   70
   35   35
Sum of absolute differences = 0
```

```
j=487
  -x1-  -x2-
  272   272
  238   238
  204   204
  170   170
  136   136
  102   102
   68    68
   34    34
Sum of absolute differences = 0

j=488
  -x1-  -x2-
  264   264
  231   231
  198   198
  165   165
  132   132
   99    99
   66    66
   33    33
Sum of absolute differences = 0

j=489
  -x1-  -x2-
  256   256
  224   224
  192   192
  160   160
  128   128
   96    96
   64    64
   32    32
Sum of absolute differences = 0

j=490
  -x1-  -x2-
  248   248
  217   217
  186   186
  155   155
  124   124
   93    93
   62    62
   31    31
Sum of absolute differences = 0

j=491
  -x1-  -x2-
  240   240
  210   210
  180   180
  150   150
  120   120
   90    90
   60    60
   30    30
Sum of absolute differences = 0

j=492
  -x1-  -x2-
  232   232
  203   203
  174   174
  145   145
  116   116
   87    87
   58    58
   29    29
Sum of absolute differences = 0

j=493
  -x1-  -x2-
  224   224
  196   196
  168   168
  140   140
  112   112
   84    84
   56    56
   28    28
Sum of absolute differences = 0

j=494
  -x1-  -x2-
  216   216
  189   189
  162   162
  135   135
  108   108
   81    81
   54    54
   27    27
Sum of absolute differences = 0

j=495
  -x1-  -x2-
  208   208
  182   182
  156   156
  130   130
  104   104
   78    78
   52    52
   26    26
Sum of absolute differences = 0

j=496
  -x1-  -x2-
  200   200
  175   175
  150   150
  125   125
  100   100
```

```
   75   75
   50   50
   25   25
Sum of absolute differences = 0

j=497
  -x1-  -x2-
  192  192
  168  168
  144  144
  120  120
   96   96
   72   72
   48   48
   24   24
Sum of absolute differences = 0

j=498
  -x1-  -x2-
  184  184
  161  161
  138  138
  115  115
   92   92
   69   69
   46   46
   23   23
Sum of absolute differences = 0

j=499
  -x1-  -x2-
  176  176
  154  154
  132  132
  110  110
   88   88
   66   66
   44   44
   22   22
Sum of absolute differences = 0

j=500
  -x1-  -x2-
  168  168
  147  147
  126  126
  105  105
   84   84
   63   63
   42   42
   21   21
Sum of absolute differences = 0

j=501
  -x1-  -x2-
  160  160
  140  140
  120  120
  100  100
   80   80
   60   60
   40   40
   20   20
Sum of absolute differences = 0

j=502
  -x1-  -x2-
  152  152
  133  133
  114  114
   95   95
   76   76
   57   57
   38   38
   19   19
Sum of absolute differences = 0

j=503
  -x1-  -x2-
  144  144
  126  126
  108  108
   90   90
   72   72
   54   54
   36   36
   18   18
Sum of absolute differences = 0

j=504
  -x1-  -x2-
  136  136
  119  119
  102  102
   85   85
   68   68
   51   51
   34   34
   17   17
Sum of absolute differences = 0

j=505
  -x1-  -x2-
  128  128
  112  112
   96   96
   80   80
   64   64
   48   48
   32   32
   16   16
Sum of absolute differences = 0

j=506
  -x1-  -x2-
  120  120
```

```
  105  105
   90   90
   75   75
   60   60
   45   45
   30   30
   15   15
Sum of absolute differences = 0

j=507
  -x1-  -x2-
  112  112
   98   98
   84   84
   70   70
   56   56
   42   42
   28   28
   14   14
Sum of absolute differences = 0

j=508
  -x1-  -x2-
  104  104
   91   91
   78   78
   65   65
   52   52
   39   39
   26   26
   13   13
Sum of absolute differences = 0

j=509
  -x1-  -x2-
   96   96
   84   84
   72   72
   60   60
   48   48
   36   36
   24   24
   12   12
Sum of absolute differences = 0

j=510
  -x1-  -x2-
   88   88
   77   77
   66   66
   55   55
   44   44
   33   33
   22   22
   11   11
Sum of absolute differences = 0

j=511
  -x1-  -x2-
   80   80
   70   70
   60   60
   50   50
   40   40
   30   30
   20   20
   10   10
Sum of absolute differences = 0

j=512
  -x1-  -x2-
   72   72
   63   63
   54   54
   45   45
   36   36
   27   27
   18   18
    9    9
Sum of absolute differences = 0
octave:63> who

*** currently compiled functions:

clock       columns    date         holes1
holes2   num2str  rem      rows

*** local user variables:

M    Np   err  j    p    x1   x2

octave:64> size(M)
ans =

  1  1

octave:65> size(x1)
ans =

  8  1

octave:66> size(err)
ans =

  512    1

octave:67> max(abs(err))
ans = 0
octave:68> 'Simple hole finding function
works!'
ans = Simple hole finding function works!
octave:69> clock
ans =
[redacted]
octave:70> diary off
```

HOLES3.M

```
function [k,h] = holes3(y,p,M);

% h = holes3(y,p,M)
% CONFIDENTIAL AND PROPRIETARY
% Edwin A. Suominen
% Finds "holes" - skipped values of set {0,1}^N in result
% of x*y mod p.
% Uses equation discovered by EAS 9/16/00

% Number of values in set S:{0,1}^N
% M = 2^N;

k = p-(M+1);

% For vector inputs...
for i=1:length(y)

  for j=1:k,

    ## Input values between M+1 and p will of necessity
    ## be mapped to holes (values not produced by inputs
    ## from set {1,2,...M} because xy mod p is a bijection
    ## (See HAC 1.8 Definition)
    ## h(j,i) = rem( (M+j)*y ,p);
    ## Equation above is simple but doesn't work when
    ## M < xy < p (which happens rarely, but it happens).

    h(j,i) = M+1 - rem(j*y(i)-k,p);
  endfor

endfor




if (nargout>=2)
  k = 1:k; k=k';
endif

endfunction
```

TEST3.M

TESTS EACH INPUT FOR ALL KEYS IN SPACE

```
## TEST3.M
## Block size is 10 bits. Input is taken from set Z:{1,2,...1024}
## Because of EAS-invented "pseudogroup" operation, output also
## falls in set Z.
## Keys are also taken from set Z - any set element is OK.

## This test proves the following:
## (1) Output set is same as input set Z.
## (2) Each input value has a unique output value, for a given
## key value.
## (3) The output value from "encrypt.m" can be converted back to
## the input value with "decrypt.m," given the key value.
## (4) For a given input value, each key value produces a unique
## output value.
## Written for Octave (GNU MATLAB alternative)

## No paging - want current screen output
page_screen_output=0;

## Set values defining set and underlying group order
N = 10; M = 2^N; # M = 1024
k = 7; p = M+k; # p = 1031 (prime)

## Create empty matrix of output values
A = zeros(M);

## Define vector with elements of set Z
v = linspace(1,M,M);

## Create string matrix of '-' neutral values for test condition codes
cc = ['-RESULTS- '; ' key: 1234']; # Header
## for each key value...
for i = 1:M
  ## insert key value before neutrals
  ccr = [num2str(i),': ----'];
  ## Leading zeros to make columns line up
  if i<10, ccr = ['0' ccr]; endif
  if i<100, ccr = ['0' ccr]; endif
  if i<1000, ccr = ['0' ccr]; endif
  cc(i+2,:) = ccr;
endfor

############ PART ONE OF TWO ##############
disp(['Tests 1-3, for each key value in set 1,2,...',num2str(M)]);
disp('-----------------------------------------------------');
```

```
## For all possible key values in Z...
for i = 1:M

  ## Show progress
  disp(['Encrypting and decrypting with key y=',num2str(i),'...']);

  ## Set key value for this iteration
  y = v(i);

  ## Encrypt all possible input values in set Z with key
  b = encrypt(v,y,N,k);
  A(:,i) = b'; # Add this output vector to output matrix

  ## Test for conditions (1),(2) now
  b = sort(b); # Sort ascending

  disp(['Output set: min=',num2str(min(b)),', max=',num2str(max(b))]);


  ##### Test Condition (1) #####
  if ( max(b)==M )
     disp('Output set is same as input set.');
    cc(i+2,7) = '+';
  else
    disp('PROBLEM: Output set larger or smaller than input set!');
    cc(i+2,7) = 'o';
  endif


  ##### Test Condition (2) #####
  ## Each input value should have a unique output value, for a given
  ## key value.
  b = diff(b); # Get differentials between sorted elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,8) = '+';
  else
    disp('PROBLEM: skipped or duplicated element(s) in output set!');
    cc(i+2,8) = 'o';
  endif


  ##### Decrypt output values for this key #####
  b = decrypt(A(:,i),y,N,k)';


  ##### Test Condition (3) #####
```

```
  ## Get differentials between plaintext-encrypted-decrypted (b) and
plaintext (v)
  b = b - v; # Should be all zeros if test passes
  if ( (max(abs(b))==0) )
    disp('All elements in input set encrypt and decrypt with key and
inverse.');
    cc(i+2,9) = '+';
  else
    disp('PROBLEM:  One  or  more  elements  do  not  match  in
encryption/decryption!');
    cc(i+2,9) = 'o';
  endif

  disp('');

endfor

############ PART TWO OF TWO ##############
disp(['Test 4, for each input value in set 1,2,...',num2str(M)]);
disp('-------------------------------------------------------');

##### Test Condition (4) #####

## For all possible input values in Z, working with full matrix of
outputs
for i = 1:M

  ## Show progress
  disp(['Analyzing outputs for input x=',num2str(i),' with all keys in
set...']);

  ## For a given input value, each key value should produce a unique
output value.
  b = diff(sort(A(i,:))); # Get differentials between sorted elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,10) = '+';
  else
    disp('Skipped or duplicated element(s) in output set.');
    cc(i+2,10) = 'o';
  endif

  disp('');

endfor

## Display test results
disp(cc)
```

RESULTS OF TEST3.M

```
octave:14> date
ans = [redacted]
octave:15> clock
ans =

[redacted]

octave:16> type test3
test3 is the script file: /1093-2/test3.m

## TEST3.M
##  Block size is 10 bits. Input is taken from set
Z:{1,2,...1024}
##  Because of EAS-invented "pseudogroup" operation,
output also
## falls in set Z.
## Keys are also taken from set Z - any set element is
OK.

## This test proves the following:
## (1) Output set is same as input set Z.
## (2) Each input value has a unique output value, for a
given
## key value.
## (3) The output value from "encrypt.m" can be converted
back to
## the input value with "decrypt.m," given the key value.
## (4) For a given input value, each key value produces a
unique
## output value.

[redacted]

## Written for Octave (GNU MATLAB alternative)

[redacted]

## No paging - want current screen output
page_screen_output=0;

## Set values defining set and underlying group order
N = 10; M = 2^N; # M = 1024
k = 7; p = M+k; # p = 1031 (prime)

## Create empty matrix of output values
A = zeros(M);

## Define vector with elements of set Z
v = linspace(1,M,M);

## Create string matrix of '-' neutral values for test
condition codes
cc = ['-RESULTS- '; ' key: 1234']; # Header
## for each key value...
for i = 1:M
  ## insert key value before neutrals
  ccr = [num2str(i),': ----'];
  ## Leading zeros to make columns line up
  if i<10, ccr = ['0' ccr]; endif
  if i<100, ccr = ['0' ccr]; endif
  if i<1000, ccr = ['0' ccr]; endif
  cc(i+2,:) = ccr;
endfor

############ PART ONE OF TWO ###############
disp(['Tests  1-3,  for  each  key  value  in  set
1,2,...',num2str(M)]);
disp('-------------------------------------------------
----');

## For all possible key values in Z...
for i = 1:M

  ## Show progress
  disp(['Encrypting  and  decrypting  with  key
y=',num2str(i),'...']);

  ## Set key value for this iteration
  y = v(i);

  ## Encrypt all possible input values in set Z with key
  b = encrypt(v,y,N,k);
  A(:,i) = b'; # Add this output vector to output matrix

  ## Test for conditions (1),(2) now
  b = sort(b); # Sort ascending

  disp(['Output  set:  min=',num2str(min(b)),',
max=',num2str(max(b))]);


  ##### Test Condition (1) #####
  if ( max(b)==M )
    disp('Output set is same as input set.');
    cc(i+2,7) = '+';
  else
    disp('PROBLEM:  Output  set  larger  or  smaller  than
input set!');
    cc(i+2,7) = 'o';
  endif


  ##### Test Condition (2) #####
  ## Each input value should have a unique output value,
for a given
  ## key value.
  b = diff(b);  # Get differentials between sorted
elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,8) = '+';
  else
    disp('PROBLEM:  skipped  or  duplicated  element(s)  in
output set!');
    cc(i+2,8) = 'o';
  endif


  ##### Decrypt output values for this key #####
  b = decrypt(A(:,i),y,N,k)';


  ##### Test Condition (3) #####
  ##  Get  differentials  between  plaintext-encrypted-
decrypted (b) and plaintext (v)
  b = b - v; # Should be all zeros if test passes
  if ( (max(abs(b))==0) )
    disp('All elements in input set encrypt and decrypt
with key and inverse.');
    cc(i+2,9) = '+';
```

```
  else
    disp('PROBLEM: One or more elements do not match in
encryption/decryption!');
    cc(i+2,9) = 'o';
  endif

  disp('');

endfor


############ PART TWO OF TWO ##############
disp(['Test  4,  for  each  input  value  in  set
1,2,...',num2str(M)]);
disp('-------------------------------------------------
----');

##### Test Condition (4) #####

## For all possible input values in Z, working with full
matrix of outputs
for i = 1:M

  ## Show progress
  disp(['Analyzing  outputs  for  input  x=',num2str(i),'
with all keys in set...']);

  ## For a given input value, each key value should
produce a unique output value.
  b = diff(sort(A(i,:))); # Get differentials between
sorted elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,10) = '+';
  else
    disp('Skipped  or  duplicated  element(s)  in  output
set.');
    cc(i+2,10) = 'o';
  endif

  disp('');

endfor

## Display test results
disp(cc)
octave:18> test3
Tests 1-3, for each key value in set 1,2,...1024
-------------------------------------------------------
Encrypting and decrypting with key y=1...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=2...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=3...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=4...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=5...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=6...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=7...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=8...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=9...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=10...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=11...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=12...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=13...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=14...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.
```

```
Encrypting and decrypting with key y=15...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=16...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=17...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=18...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=19...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=20...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=21...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=22...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=23...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=24...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=25...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=26...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=27...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=28...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=29...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=30...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=31...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=32...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=33...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=34...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=35...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
```

```
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1014...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1015...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1016...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1017...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1018...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1019...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1020...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1021...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1022...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1023...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1024...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Test 4, for each input value in set 1,2,...1024
-------------------------------------------------------
Analyzing outputs for input x=1 with all keys in set...
All elements in output set are unique.

Analyzing outputs for input x=2 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=3 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=4 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=5 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=6 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=7 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=8 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=9 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=10 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=11 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=12 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=13 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=14 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=15 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=16 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=17 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=18 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=19 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=20 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=21 with all keys in set...
```

```
Analyzing outputs for input x=995 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=996 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=997 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=998 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=999 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1000 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1001 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1002 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1003 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1004 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1005 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1006 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1007 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1008 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1009 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1010 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1011 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1012 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1013 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1014 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1015 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1016 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1017 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1018 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1019 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1020 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1021 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1022 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1023 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1024 with all keys in
set...
Skipped or duplicated element(s) in output set.

-RESULTS-
 key: 1234
0001: ++++
0002: +++o
0003: +++o
0004: +++o
0005: +++o
0006: +++o
0007: +++o
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0009: +++o
0010: +++o
0011: +++o
0012: +++o
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0025: +++o
0026: +++o
0027: +++o
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0099: +++o
0100: +++o
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0975: +++o
0976: +++o
0977: +++o
0978: +++o
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1007: +++o
1008: +++o
1009: +++o
1010: +++o
1011: +++o
1012: +++o
1013: +++o
1014: +++o
1015: +++o
1016: +++o
1017: +++o
1018: +++o
1019: +++o
1020: +++o
1021: +++o
1022: +++o
1023: +++o
1024: +++o
octave:19> diary off
```

RESULTS OF TEST3B.M

```
octave:4> date
ans = [redacted]
octave:5> clock
ans =
  [redacted]

octave:6> type test3b
test3b is the script file: /1093-2/test3b.m

## TEST3B.M
## Block size is 10 bits. Input is taken from set Z:{1,2,...1024}
## Because of EAS-invented "pseudogroup" operation, output also
## falls in set Z.
## Keys are also taken from set Z - any set element is OK.

## This test analyzes outputs for a given input over all
## possible keys.

[redacted]
## Written for Octave (GNU MATLAB alternative)
[redacted]

## No paging - want current screen output
page_screen_output=0;

## Set values defining set and underlying group order
N = 10; M = 2^N; # M = 1024
k = 7; p = M+k; # p = 1031 (prime)

## Define vector with elements of set Z
v = linspace(1,M,M);

## Define vector of skip/repeat counts
cc = zeros(1,M);

disp(['Test for each input value in set 1,2,...',num2str(M)]);
disp('------------------------------------------------------');

## For all possible input values in Z...
for i = 1:M

  ## Show progress
  disp(['Encrypting with input value y=',num2str(i),'...']);

  ## Set input value for this iteration
  x = v(i);

  ## Encrypt input value with all keys in set Z
  for j = 1:M
    b(j) = encrypt(x,v(j),N,k);
  endfor
```

```
  disp(['Output set: min=',num2str(min(b)),', max=',num2str(max(b))]);
  disp('');

  ## Identify any skipped or repeated set elements
  ## with vector of index numbers
  b1 = sort(b); # Sort ascending
  b2 = [diff(b1)']; # Should be all 1's...
  b2 = b2~=1; # ...so 1's indicate skips/repeats

  Nsr = sum(b2); # Count of skips/repeats

  b3 = b2 .* v(1:M-1); # map index numbers to skips/repeats
  b3 = sort(b3); # Sort ascending
  b4 = b3(M-Nsr:M-1); # Select only skips/repeats

  if (Nsr > 0)

    disp(['There are ',num2str(Nsr),' skips & repeats, at:');
    disp(b4);
    disp('------------------------------------------------------');

    c = zeros(6,Nsr); # Start with empty ("0") matrix
    for j = 1:Nsr
      k1 = max([1 b4(j)-2]);
      k2 = min([b4(j)+3 M]);
      c(1:k2-k1+1,j) = b1(k1:k2);
    endfor

    disp(c)

  endif

  cc(i) = Nsr; # Add this count to vector
  disp(['Maximum skips & repeats for a given input (so far):',num2str(max(cc))]);
  disp('');

endfor

. . .

octave:9> test3b
Test for each input value in set 1,2,...1024
------------------------------------------------------
Encrypting with input value y=1...
Output set: min=1, max=1024

Maximum skips & repeats for a given input (so far):0

Encrypting with input value y=2...
Output set: min=1, max=1024

There are 6 skips & repeats, at:
    10   518   520  1021  1022  1023
------------------------------------------------------
     8   515   517  1016  1017  1018
     9   516   517  1017  1018  1020
    10   517   518  1018  1020  1022
    10   517   518  1020  1022  1024
```

```
    11   518   519  1022  1024     0
    12   518   520  1024     0     0
Maximum skips & repeats for a given input (so far):6

Encrypting with input value y=3...
Output set: min=1, max=1024

There are 8 skips & repeats, at:
    10   346   690   692  1016  1018  1020  1022
-------------------------------------------------------
     8   343   686   688  1010  1012  1015  1018
     9   344   687   688  1011  1014  1017  1020
    10   345   688   689  1012  1015  1018  1021
    10   345   688   689  1014  1017  1020  1023
    11   346   689   690  1015  1018  1021  1024
    12   347   689   691  1017  1020  1023     0
Maximum skips & repeats for a given input (so far):8

Encrypting with input value y=4...
Output set: min=1, max=1024

There are 10 skips & repeats, at:
     3   260   519   523   778  1011  1014  1017  1020  1023
-----------------------------------------------------------
     1   257   515   518   772  1004  1008  1012  1016  1020
     2   258   516   519   773  1005  1009  1013  1017  1021
     3   259   517   520   774  1006  1010  1014  1018  1022
     3   259   517   520   774  1008  1012  1016  1020  1024
     4   260   518   521   775  1009  1013  1017  1021     0
     5   261   519   522   776  1010  1014  1018  1022     0
Maximum skips & repeats for a given input (so far):10

Encrypting with input value y=5...
Output set: min=1, max=1024

There are 10 skips & repeats, at:
     6   212   418   624   830  1005  1009  1013  1017  1021
-----------------------------------------------------------
     4   209   414   619   824   998  1003  1008  1013  1018
     5   210   415   620   825   999  1004  1009  1014  1019
     6   211   416   621   826  1000  1005  1010  1015  1020
     6   211   416   621   826  1002  1007  1012  1017  1022
     7   212   417   622   827  1003  1008  1013  1018  1023
     8   213   418   623   828  1004  1009  1014  1019  1024
Maximum skips & repeats for a given input (so far):10

Encrypting with input value y=6...
Output set: min=1, max=1024

There are 10 skips & repeats, at:
   176   346   518   691   864   999  1004  1009  1014  1019
-----------------------------------------------------------
   174   343   514   686   858   992   998  1004  1010  1016
   175   344   515   687   859   993   999  1005  1011  1017
   176   345   516   688   860   994  1000  1006  1012  1018
   176   345   516   688   860   996  1002  1008  1014  1020
   177   346   517   689   861   997  1003  1009  1015  1021
   178   347   518   690   862   998  1004  1010  1016  1022
```

```
Maximum skips & repeats for a given input (so far):10

Encrypting with input value y=7...
Output set: min=1, max=1023

There are 11 skips & repeats, at:
   149   297   445   593   741   889   994  1000  1006  1012  1018
-------------------------------------------------------------------
   147   294   441   588   735   882   986   993  1000  1007  1014
   148   295   442   589   736   883   987   994  1001  1008  1015
   149   296   443   590   737   884   988   995  1002  1009  1016
   149   296   443   590   737   884   990   997  1004  1011  1018
   150   297   444   591   738   885   991   998  1005  1012  1019
   151   298   445   592   739   886   992   999  1006  1013  1020
Maximum skips & repeats for a given input (so far):11

Encrypting with input value y=8...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
     3     4   261   520   521   779   988   995  1002  1009  1016  1023
-------------------------------------------------------------------------
     1     2   257   515   516   772   980   988   996  1004  1012  1020
     2     3   258   516   517   773   981   989   997  1005  1013  1021
     3     3   259   517   517   774   982   990   998  1006  1014  1022
     3     3   259   517   517   774   984   992  1000  1008  1016  1024
     3     4   260   517   518   775   985   993  1001  1009  1017     0
     4     5   261   518   519   776   986   994  1002  1010  1018     0
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=9...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
     2   346   347   464   692   922   982   990   998  1006  1014  1022
-------------------------------------------------------------------------
     1   343   344   459   686   915   974   983   992  1001  1010  1019
     2   344   345   460   687   916   975   984   993  1002  1011  1020
     2   345   345   461   688   917   976   985   994  1003  1012  1021
     3   345   345   461   688   917   978   987   996  1005  1014  1023
     4   345   346   462   689   918   979   988   997  1006  1015  1024
     0   346   347   463   690   919   980   989   998  1007  1016     0
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=10...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
     2     4   416   520   829   933   976   985   994  1003  1012  1021
-------------------------------------------------------------------------
     1     2   412   515   823   926   968   978   988   998  1008  1018
     2     2   413   516   824   927   969   979   989   999  1009  1019
     2     3   414   517   825   928   970   980   990  1000  1010  1020
     3     3   414   517   825   928   972   982   992  1002  1012  1022
     3     4   415   518   826   929   973   983   993  1003  1013  1023
     0     5   416   519   827   930   974   984   994  1004  1014  1024
Maximum skips & repeats for a given input (so far):12
```

```
   17   35   53   71   89  107  114  342  458  687  687  859
   19   37   55   73   91  109  114  342  458  687  687  859
   20   38   56   74   92  110  115  343  459  687  688  860
   21   39   57   75   93  111  116  344  460  688  689  861
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1014...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   16   32   48   64   80   96  236  479  661  845  907  969
-------------------------------------------------------------
   14   31   48   65   82   99  240  482  663  846  907  968
   15   32   49   66   83  100  241  483  664  847  908  969
   16   33   50   67   84  101  242  484  665  848  909  970
   18   35   52   69   86  103  242  484  665  848  909  970
   19   36   53   70   87  104  243  485  666  849  910  971
   20   37   54   71   88  105  244  486  667  850  911  972
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1015...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   15   30   45   60   75   90  121  188  511  641  771  901
-------------------------------------------------------------
   13   29   45   61   77   93  125  191  513  642  771  900
   14   30   46   62   78   94  126  192  514  643  772  901
   15   31   47   63   79   95  127  193  515  644  773  902
   17   33   49   65   81   97  127  193  515  644  773  902
   18   34   50   66   82   98  128  194  516  645  774  903
   19   35   51   67   83   99  129  195  517  646  775  904
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1016...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   14   28   42   56   70   84  199  338  614  752  891  961
-------------------------------------------------------------
   12   27   42   57   72   87  203  341  616  753  891  960
   13   28   43   58   73   88  204  342  617  754  892  961
   14   29   44   59   74   89  205  343  618  755  893  962
   16   31   46   61   76   91  205  343  618  755  893  962
   17   32   47   62   77   92  206  344  619  756  894  963
   18   33   48   63   78   93  207  345  620  757  895  964
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1017...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   13   26   39   52   65   78  140  436  437  658  734  956
-------------------------------------------------------------
   11   25   39   53   67   81  144  439  440  659  734  955
   12   26   40   54   68   82  145  440  441  660  735  956
   13   27   41   55   69   83  146  441  441  661  736  957
   15   29   43   57   71   85  146  441  441  661  736  957
```

```
   16   30   44   58   72   86  147  441  442  662  737  958
   17   31   45   59   73   87  148  442  443  663  738  959
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1018...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   12   24   36   48   60   72   73  231  233  631  632  871
------------------------------------------------------------
   10   23   36   49   62   75   76  234  236  632  633  870
   11   24   37   50   63   76   77  235  236  633  634  871
   12   25   38   51   64   77   77  236  237  634  634  872
   14   27   40   53   66   77   79  236  237  634  634  872
   15   28   41   54   67   79   80  237  238  634  635  873
   16   29   42   55   68   80   81  237  239  635  636  874
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1019...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   11   22   33   44   55   66  336  508  682  769  857  944
------------------------------------------------------------
    9   21   33   45   57   69  340  511  684  770  857  943
   10   22   34   46   58   70  341  512  685  771  858  944
   11   23   35   47   59   71  342  513  686  772  859  945
   13   25   37   49   61   73  342  513  686  772  859  945
   14   26   38   50   62   74  343  514  687  773  860  946
   15   27   39   51   63   75  344  515  688  774  861  947
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1020...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   10   20   30   40   50   60   85  182  275  652  747  936
------------------------------------------------------------
    8   19   30   41   52   63   89  185  277  653  747  935
    9   20   31   42   53   64   90  186  278  654  748  936
   10   21   32   43   54   65   91  187  279  655  749  937
   12   23   34   45   56   67   91  187  279  655  749  937
   13   24   35   46   57   68   92  188  280  656  750  938
   14   25   36   47   58   69   93  189  281  657  751  939
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1021...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
    9   18   27   36   45   54  199  407  510  615  616  823
------------------------------------------------------------
    7   17   27   37   47   57  203  410  512  616  617  822
    8   18   28   38   48   58  204  411  513  617  618  823
    9   19   29   39   49   59  205  412  514  618  618  824
   11   21   31   41   51   61  205  412  514  618  618  824
   12   22   32   42   52   62  206  413  515  618  619  825
   13   23   33   43   53   63  207  414  516  619  620  826
```

```
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1022...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
    8   16   24   32   40   48  222  336  567  684  799  915
-------------------------------------------------------------
    6   15   24   33   42   51  226  339  569  685  799  914
    7   16   25   34   43   52  227  340  570  686  800  915
    8   17   26   35   44   53  228  341  571  687  801  916
   10   19   28   37   46   55  228  341  571  687  801  916
   11   20   29   38   47   56  229  342  572  688  802  917
   12   21   30   39   48   57  230  343  573  689  803  918
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1023...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
    7   14   21   28   35   42  248  249  511  641  771  901
-------------------------------------------------------------
    5   13   21   29   37   45  252  253  513  642  771  900
    6   14   22   30   38   46  253  254  514  643  772  901
    7   15   23   31   39   47  254  254  515  644  773  902
    9   17   25   33   41   49  254  254  515  644  773  902
   10   18   26   34   42   50  254  255  516  645  774  903
   11   19   27   35   43   51  255  256  517  646  775  904
Maximum skips & repeats for a given input (so far):12

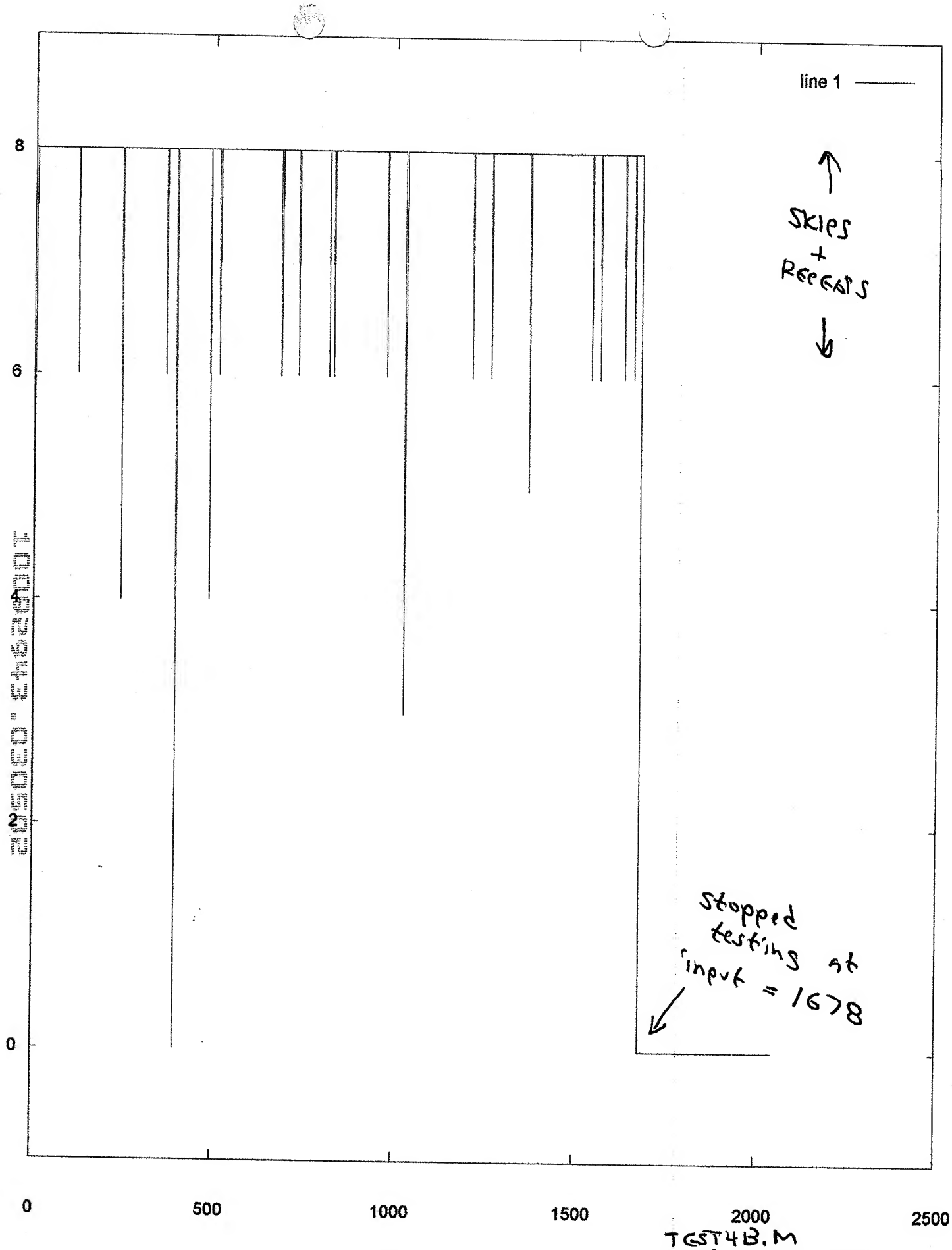
Encrypting with input value y=1024...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
    6   12   18   24   30   36  137  286  435  584  733  882
-------------------------------------------------------------
    4   11   18   25   32   39  141  289  437  585  733  881
    5   12   19   26   33   40  142  290  438  586  734  882
    6   13   20   27   34   41  143  291  439  587  735  883
    8   15   22   29   36   43  143  291  439  587  735  883
    9   16   23   30   37   44  144  292  440  588  736  884
   10   17   24   31   38   45  145  293  441  589  737  885
Maximum skips & repeats for a given input (so far):12

octave:10> date
ans = [redacted]
octave:11> clock
ans =

[redacted]

octave:12> diary off
```



TGST4B.M
WITH M = 2048
p = 2053

W-1

Pronounceable Passphrase Worksheet

by Edwin A. Suominen

Digit Content	Min. Digits	Entropy	Example Passphrase
Alternating consonants (C), Vowels (V)	C: 11	About 64 bits, for	nihudezo dogiz pozubume
C,13: {b,d,g,h,k,l,m,n,p,r,s,t,z} V,5: {a,e,i,o,u}	V: 10	minimum digits	

The vowels and phonetically distinct consonanants below are pseudorandomly distributed, created using alternating pseudorandom lookups to a list of consonants b,d,g,h,k,l,m,n,p,r,s,t,z and vowels a,e,i,o,u. Alternating randomly selected consonants and randomly selected vowels from the array below form passphrases that have a faintly Oriental or African sound to them, and are more memorable than random alphanumerics. Consonants c,f,j,q,v,w,x,y are omitted because the passphrases tend to have a more distinct sound and are easier to pronounce without them.

You should split the consonant/vowel pairs into groups to make the passphrase pronounceable and thus more memorable. The suggested way of grouping the minimum 11 consonants and 10 vowels is as follows: CVCVCVCV CVCVC CVCVCVCV. Note that the middle group begins and ends with a consonant. The resulting passphrase has a distinct sound that makes you wonder if the "words" show up in some foreign language even though they're just groups of randomly chosen letters.

Unbend a paper clip slightly, repeatedly toss the clip onto a printout of this page without aiming it anywhere in particular, and select the consonant/vowel pair to which the unbent end comes closest to get the next digit in your consonant-vowel sequence. Don't use both digits from a pair – each digit in your passphrase needs its own toss. With good random tosses, you can expect the clip to bounce outside the array of digits below about half the time. Just toss again. Don't aim at any particular region.

ne	ze	nu	hi	ni	ba	hu	pi	pu	lu	me	ki	lu	ge	le	mu	nu	ko	se	ze	ta	ba	ga	ro	ta	be	ko	te
lo	ru	hu	gi	ra	ga	ro	do	ro	zu	tu	si	mu	pu	lo	bi	ze	tu	ha	ti	ru	to	ni	ze	bu	be	si	ma
ro	le	hi	ni	bo	ma	do	hu	ma	lu	bu	zi	ru	sa	ta	zo	po	go	ta	hi	ri	bi	te	la	me	go	ho	si
go	ka	hi	bo	li	ro	mi	ta	mo	ki	ku	hu	ri	le	da	ra	za	mi	le	mi	da	ra	zu	ki	ke	bi	mi	do
mo	ta	tu	ta	ra	si	la	go	ki	su	ki	mi	re	ba	le	so	to	za	ba	te	hi	ri	da	go	za	ka	sa	za
mo	na	la	me	ku	bu	mu	me	no	ke	ri	be	ho	tu	mu	ki	no	re	ho	pe	mi	hu	ho	pi	ge	ro	to	se
go	ka	ze	ri	du	sa	de	bi	si	de	mi	pi	se	po	le	mi	bu	re	pe	du	so	lu	la	ri	mo	mo	bo	go
to	ge	ne	go	lo	ru	sa	da	ma	ga	ni	pe	si	se	bu	di	pi	da	pu	ta	la	ku	hu	ko	go	za	si	de
tu	me	po	ro	to	pe	ku	hu	ra	ha	bu	zo	ti	na	ba	ma	se	te	pi	do	to	ke	so	do	re	ru	ze	ru
to	lo	ne	ki	ta	ha	go	go	li	ra	po	de	na	du	ba	su	po	ka	lu	bu	hi	be	za	la	ke	bo	ne	mi
he	ni	te	ma	ni	pu	ho	ru	ne	da	nu	de	ge	ge	so	ge	lu	re	ho	bi	po	li	ma	ri	su	pa	te	du
zi	ta	ma	lu	ti	bu	ha	ba	hu	mu	su	za	ko	to	ga	te	tu	ru	la	ki	ru	ze	po	ni	pi	ho	sa	pa
pa	bu	pi	za	du	go	ga	su	mu	re	su	ni	re	hi	bi	ko	po	la	zu	ri	ku	ka	zi	ba	pa	tu	di	da
ma	mo	di	hu	po	ro	ho	ku	se	pa	bo	la	ga	ne	me	pi	pu	gu	he	li	ha	gi	bo	he	hu	so	ru	pa
se	be	pe	ga	se	ni	lo	bo	se	ka	be	gi	de	zi	di	me	mu	lu	pe	ku	tu	go	pe	hi	di	bi	de	ne
mo	mi	ki	ni	ka	ha	nu	ka	za	do	he	ni	da	du	ri	se	du	lo	se	tu	pe	to	ti	hu	na	ru	se	ta
na	ho	bu	tu	ze	ge	ru	ru	ba	ga	li	ro	so	mu	du	he	da	go	mu	da	ne	tu	mu	pi	ne	bu	ka	la
li	le	he	me	gi	ru	ge	hi	lo	po	ho	pe	no	mu	pe	ta	lo	re	na	di	gi	ko	ze	re	pi	ga	ba	ki
ke	su	zo	ra	no	di	lu	du	be	za	na	po	do	su	ra	do	si	he	me	bi	ga	ra	ha	do	za	ru	ku	ri
zi	bu	do	lo	ga	ki	ha	zi	ka	ru	re	ke	ti	lo	zu	zu	gu	bu	ma	bo	hi	he	si	du	ne	hu	da	ka
te	ne	na	zi	hu	si	da	no	mo	te	bu	ko	tu	ro	mu	ne	ma	su	li	ba	ga	te	se	mo	bi	re	si	tu
ta	pe	bo	bu	ma	tu	ge	be	ri	no	pa	do	ri	za	ra	ho	so	to	da	ze	ze	lo	he	mi	ha	la	de	me
pu	ha	no	te	de	la	zi	hu	gi	ro	te	pa	mo	bu	mu	pe	da	sa	pe	na	pi	ti	ru	ro	pi	go	li	ku
gi	gi	pu	za	pe	go	ti	lo	pu	du	su	mo	ta	ki	ha	ge	ka	ke	pa	tu	po	hi	be	lu	sa	bo	li	so
ko	ra	ri	zi	po	ra	la	ze	pe	ru	zu	ra	mu	zu	du	ka	lo	ku	zo	po	mo	no	he	ze	su	ho	le	gi
te	go	pu	ka	ga	la	ki	lu	pa	ti	na	ga	gi	ba	ka	pi	ka	su	gi	mo	no	si	te	li	gi	se	zi	ze
ze	za	lu	ge	ro	zi	ra	do	bo	ma	zo	mi	ke	ze	la	bu	ru	be	ki	te	ga	ni	go	go	gi	ga	no	ha
si	bu	be	mi	ta	pa	de	ke	ta	lu	ru	ma	ra	si	se	te	me	pe	pu	gi	mu	me	li	bi	bi	hi	zu	me
de	he	ku	zu	nu	gi	pi	zi	ga	ka	ze	lo	re	ru	ha	zu	ta	ku	pi	po	me	la	da	mu	li	de	he	be
pi	le	mu	ko	ki	nu	ke	ga	hu	la	la	ri	po	su	ri	na	ru	no	ta	ro	ho	zi	so	ru	ri	go	su	hu
gi	ra	ru	pu	mu	gi	le	ga	ti	ne	go	le	ba	da	gi	si	si	ni	ra	zu	ma	ha	bu	le	se	ze	ro	go
pi	no	bu	ne	nu	ma	si	re	ka	gu	zo	si	bo	sa	be	zo	si	ko	po	li	gi	ra	pe	ne	gu	ri	la	ki
go	ka	ze	ne	ro	mo	bi	nu	he	no	hu	ne	ha	lo	ne	le	hu	hi	so	du	de	li	ta	ze	li	hu	de	do
ga	ha	bi	he	su	ku	pe	de	he	hu	ta	so	zo	he	bu	ti	hi	gi	ta	ti	ze	pe	gu	ni	ka	ra	su	pi
lu	ge	pe	ka	pu	na	da	mu	ti	du	mo	pi	de	pu	pa	ri	si	bi	sa	mi	li	ko	mi	be	ge	ku	su	po
me	mi	tu	ze	ge	ki	gu	hu	hi	lo	tu	da	mu	to	ru	gi	ru	ra	mo	na	ma	ba	si	se	ri	ro	ro	ka
po	ge	do	sa	pu	ma	so	na	ba	la	li	ka	ne	ba	di	te	zo	so	ru	mi	se	ni	gu	na	ta	di	zi	ri
ga	ga	ri	no	ho	sa	ne	mo	le	hi	he	re	ta	ne	go	ki	de	pi	de	gi	mu	du	tu	bo	to	ki	po	di
ma	ge	bo	se	ne	hi	ni	ma	go	li	le	ra	ri	mu	si	lo	gi	zo	ku	bo	ro	ze	po	to	do	go	re	tu
ga	ka	la	ki	za	ti	me	te	zi	ho	mi	ta	no	ne	lo	za	ma	mo	te	si	sa	ba	da	ku	ko	be	ki	go
zi	zu	tu	pe	ma	ba	zu	lo	ra	ge	hu	pa	te	ra	ra	ko	zu	ni	se	he	to	ma	se	gu	gu	ba	bi	te
pe	zo	ne	no	gu	ne	bi	ga	ra	na	ra	na	la	la	do	nu	du	za	ro	li	li	mu	ne	ru	ni	he	te	ma

20SOEO" Eh626007

Description	Digit Space #1	No. of #1 Digits	Entropy #1 (bits)	Digit Space #2	No. of #2 Digits	Entropy #2 (bits)	Digit Space #3	No. of #3 Digits	Entropy #3 (bits)	Total Entropy
6 word DICEWARE (http://diceware.com)	7776	6	77.54888	1	0	0	1	0	0	**77.5**
14 alphanumeric digits except lowercase "L" (can group 5-4-5 or 4-6-4)	35	14	71.80996	1	0	0	1	0	0	**71.8**
13 alphanumeric digits except lowercase "L" (can group 4-5-4)	35	13	66.68068	1	0	0	1	0	0	**66.7**
Pairs of phonetically distinct consonants "b,d,g,h,k,l,m,n,p,r,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	13	11	40.70484	5	10	23.21928	1	0	0	**63.9**
12 alphanumeric digits except lowercase "L" separated randomly into three groups	35	12	61.5514	11	1	3.459432	1	0	0	**65.0**
5 word DICEWARE	7776	5	64.62406	1	0	0	1	0	0	**64.6**
12 alphanumeric digits except lowercase "L" (can be grouped 4-4-4)	35	12	61.5514	1	0	0	1	0	0	**61.6**
4 words from 70K dictionary	70000	4	64.38027	1	0	0	1	0	0	**64.4**
Three dates, Month, Day, Year	12	3	10.75489	30	3	14.72067	500	3	26.89735	**52.4**
Random first name (10K) plus M.I. Plus last name (50K)	10000	1	13.28771	50000	1	15.60964	26	1	4.70044	**33.6**
Random street address	20000	1	14.28771	8	1	3	10000	1	13.28771	30.6
++	No.			NSEW		30.6	20	1	4.321928	**34.9**
(e.g.,	1	0	0	1	0	0	1	0	0	**0.0**
Passwords for smartcards)	1	0	0	1	0	0	1	0	0	**0.0**
Pairs of phonetically distinct consonants "b,d,g,h,k,l,m,n,p,r,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	13	4	14.80176	5	4	9.287712	1	0	0	**24.1**
5 alphanumeric digits except lowercase "L"	35	5	25.64642	1	0	0	1	0	0	**25.6**
(e.g.,	1	0	0	1	0	0	1	0	0	**0.0**
Medium Security for use with secure delay)	1	0	0	1	0	0	1	0	0	**0.0**
Pairs of phonetically distinct consonants "b,d,g,h,k,l,m,n,p,r,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	13	8	29.60352	5	8	18.57542	1	0	0	**48.2**
Pairs of phonetically distinct consonants "b,d,g,h,k,n,p,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	10	8	26.57542	5	8	18.57542	1	0	0	**45.2**
Groups of numbers 1000-9192 = {0-8192}+1000	8192	4	52	1	0	0	1	0	0	52
	1	0	0	1	0	0	1	0	0	**0.0**
	1	0	0	1	0	0	1	0	0	**0.0**

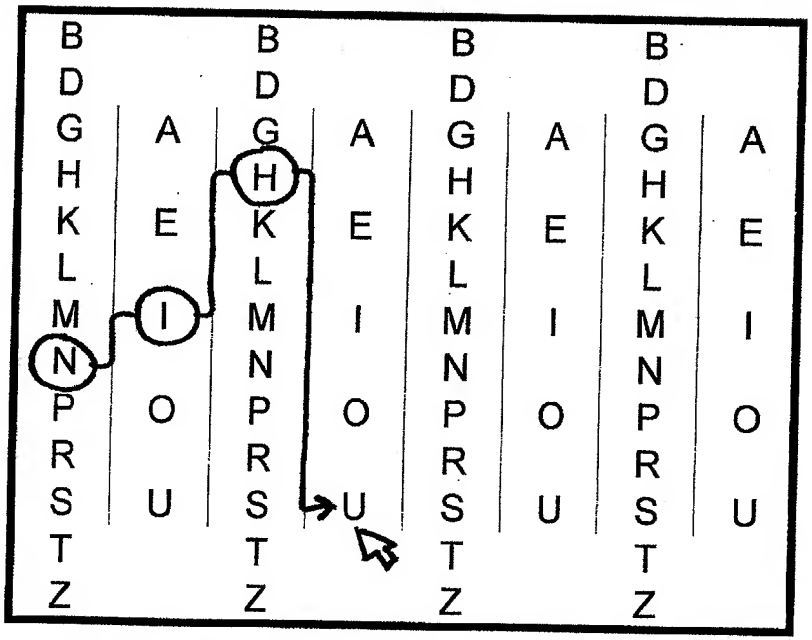
SECURE PASSPHRASE

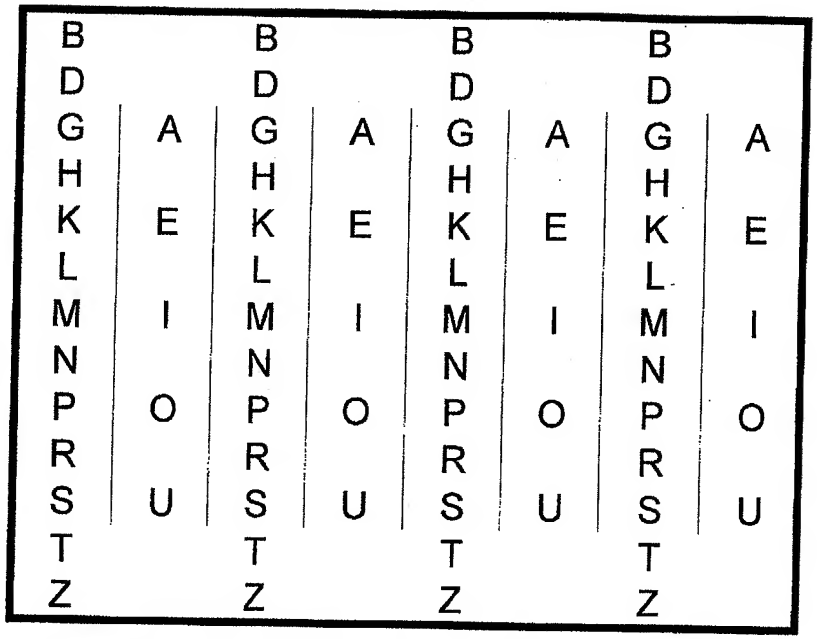
Z2-Z6 illustrate screen shots of a secure passphrase entry system according to various aspects of the invention, illustrating an exemplary user interface at different points during the input of a passphrase without the use of keystrokes. Thus, the security hazard of keystroke loggers can be avoided. In addition, the mouse-based input method may be preferred by users over the use of a keyboard, for example when they are entering their passphrase to browse encrypted e-mails or files. In an experiment the applicant carried out, "entering" the passphrase by the mouse input method (simulated by tapping a pen onto a printout similar to Z2-Z6) did not take him much longer than typing in the passphrase.

Advantageously, the passphrase is represented in the illustrated embodiment (as it is entered) both as circled letters and has a pair of stair-stepped line segments having characteristic shapes. Viewing the passphrase and its associated characteristic shapes of the line segments helps the user to remember the passphrase. Human brains are good at remembering pronounceable words (even when they are nonsense words) and are also good at remembering characteristic shapes. The combination of both characteristics of a unique passphrase can be expected to improve the user's ability to remember it when the time comes to input the passphrase.

A delay system according to another aspect of the invention, illustrated in the block diagrams of Z7 and Z8, makes a secure delay according to various aspects of the invention less unobtrusive to the user. It does so by beginning the delay process when the passphrase has been partially entered. Advantageously, such a system performs the delay computations substantially in parallel with the unavoidable delay of the user's input of the passphrase. Even when typing quickly, it took the applicant at least about three seconds to enter the passphrase during his experiment. This is a substantial period of delay that, when made computationally unavoidable, makes cracking the 2^48 possible combinations of the randomly chosen passphrase nearly impossible with the computing horsepower available around the date of filing of the present application. (See Z9 and Z10 for a detailed computational analysis.) The screen shots of Z2-Z6 show the "private key delayed unlocking" beginning with the first consonant-vowel pair entered by the user. The delayed unlocking (the inventive "computationally unavoidable" delay) continues substantially in parallel with the user's input of additional consonant-vowel pairs. Note Z6, in which the passphrase is confirmed and the private key has been completely unlocked.

###



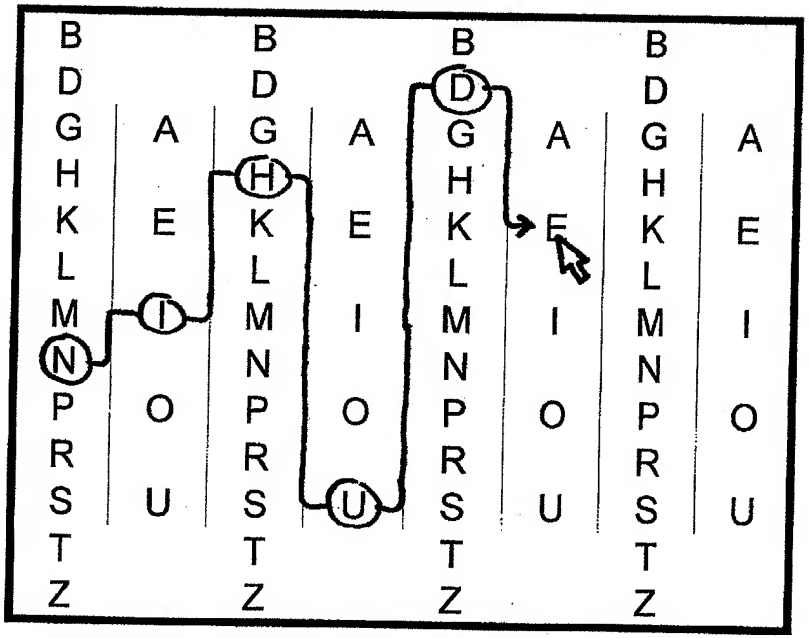


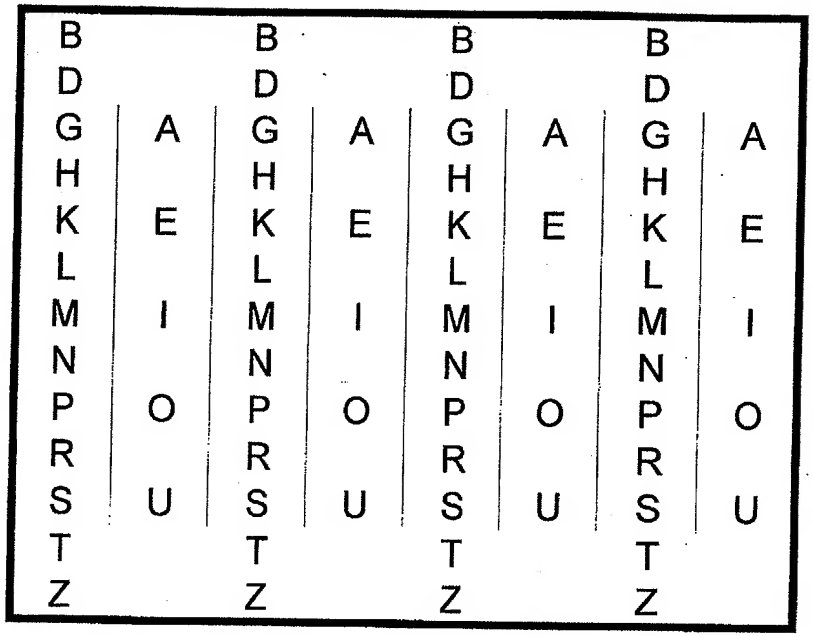
N I H __ c v c v c v c v c v c v

Click on the first letter of your passphrase and move your mouse pointer up and down over the next column of letters to move the arrow to the second letter of the passphrase. Then click and repeat until you've entered the entire passphrase. If you prefer, you can drag the arrow line thorough all letters of the passphrase without releasing your mouse button.

If you are certain your computer is secure and your keystrokes are not being logged somehow, you can also type your passphrase in the text box above.

Private Key Delayed Unlock



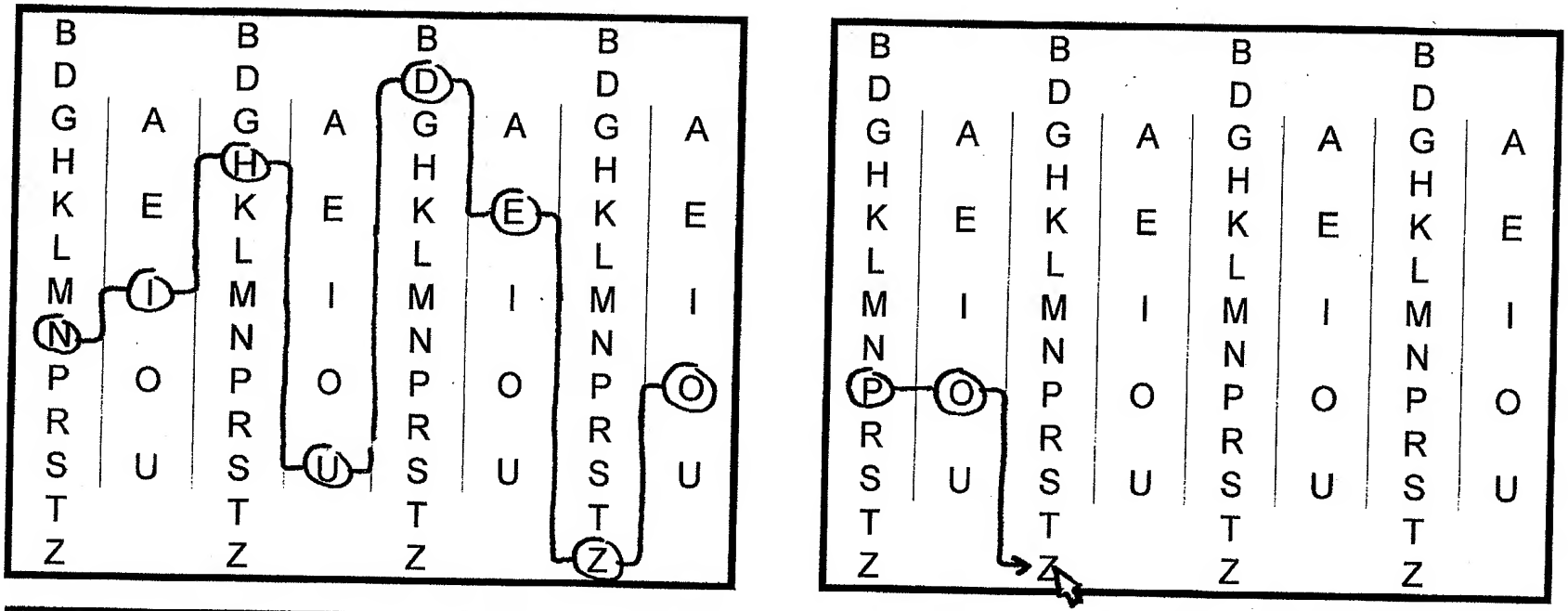


N	I	H	U	D	_	c	v	c	v	c	v	c	v	c	v

Click on the first letter of your passphrase and move your mouse pointer up and down over the next column of letters to move the arrow to the second letter of the passphrase. Then click and repeat until you've entered the entire passphrase. If you prefer, you can drag the arrow line thorough all letters of the passphrase without releasing your mouse button.

If you are certain your computer is secure and your keystrokes are not being logged somehow, you can also type your passphrase in the text box above.

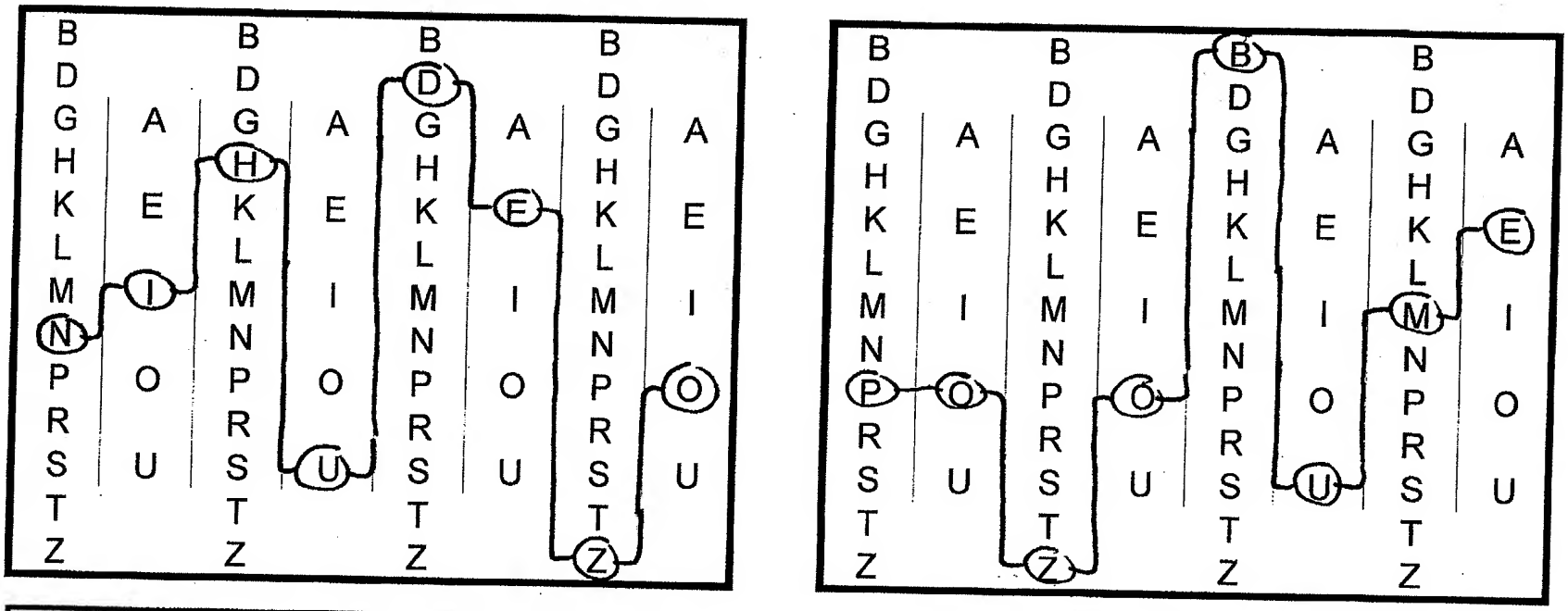
Private Key Delayed Unlock



Click on the first letter of your passphrase and move your mouse pointer up and down over the next column of letters to move the arrow to the second letter of the passphrase. Then click and repeat until you've entered the entire passphrase. If you prefer, you can drag the arrow line thorough all letters of the passphrase without releasing your mouse button.

If you are certain your computer is secure and your keystrokes are not being logged somehow, you can also type your passphrase in the text box above.

Private Key Delayed Unlock



Click on the first letter of your passphrase and move your mouse pointer up and down over the next column of letters to move the arrow to the second letter of the passphrase. Then click and repeat until you've entered the entire passphrase. If you prefer, you can drag the arrow line thorough all letters of the passphrase without releasing your mouse button.

If you are certain your computer is secure and your keystrokes are not being logged somehow, you can also type your passphrase in the text box above.

Private Key Delayed Unlock

[redacted]

B		B		B		B	
D		D		D		D	
G	A	G	A	G	A	G	A
H		H		H		H	
K	E	K	E	K	E	K	E
L		L		L		L	
M	I	M	I	M	I	M	I
N		N		N		N	
P	O	P	O	P	O	P	O
R		R		R		R	
S	U	S	U	S	U	S	U
T		T		T		T	
Z		Z		Z		Z	

B		B		B		B	
D		D		D		D	
G	A	G	A	G	A	G	A
H		H		H		H	
K	E	K	E	K	E	K	E
L		L		L		L	
M	I	M	I	M	I	M	I
N		N		N		N	
P	O	P	O	P	O	P	O
R		R		R		R	
S	U	S	U	S	U	S	U
T		T		T		T	
Z		Z		Z		Z	

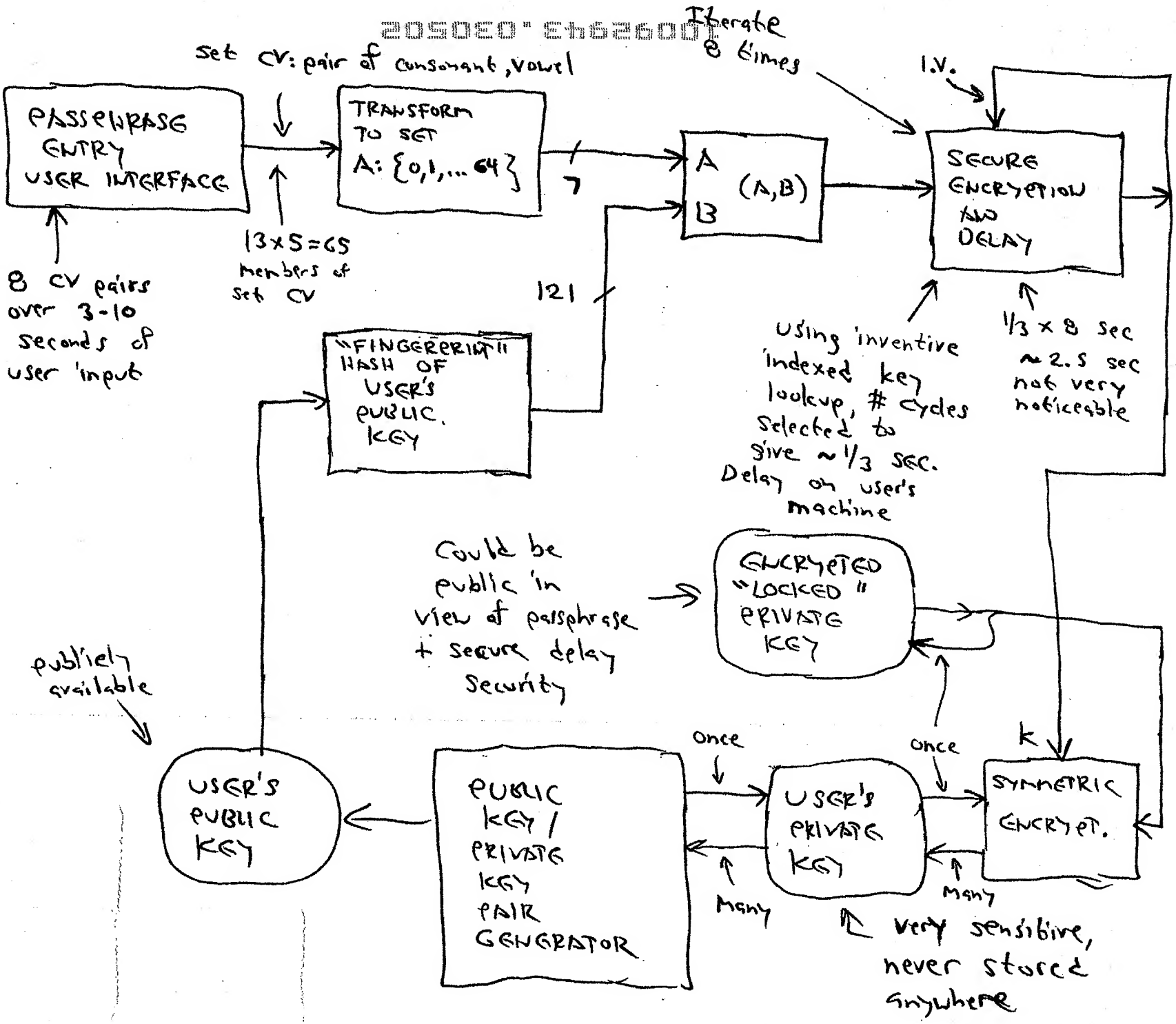
Passphrase confirmed. Your signature has been applied.

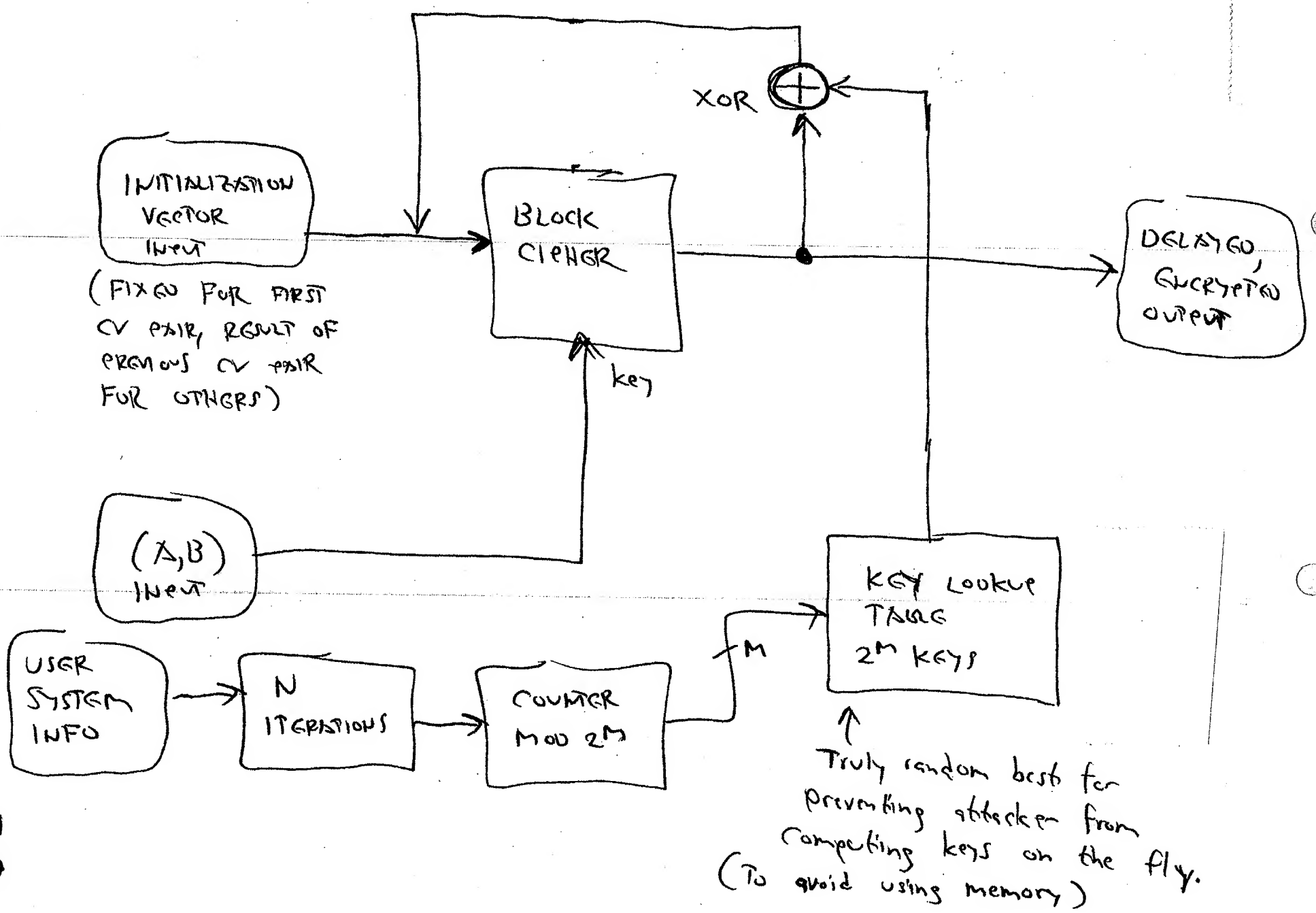
Click on the first letter of your passphrase and move your mouse pointer up and down over the next column of letters to move the arrow to the second letter of the passphrase. Then click and repeat until you've entered the entire passphrase. If you prefer, you can drag the arrow line thorough all letters of the passphrase without releasing your mouse button.

If you are certain your computer is secure and your keystrokes are not being logged somehow, you can also type your passphrase in the text box above.

Private Key Delayed Unlock

[redacted]


Iterate 8 times
Set CV: pair of consonant, vowel
I.V.
PASSPHRASE ENTRY USER INTERFACE
TRANSFORM TO SET A: {0,1,... 64}
7
A
B
(A,B)
SECURE ENCRYPTION AND DELAY
8 CV pairs over 3-10 seconds of user input
13 x 5 = 65 members of set CV
121
"FINGERPRINT" HASH OF USER'S PUBLIC KEY
Using inventive indexed key lookup, # cycles selected to give ~1/3 sec. Delay on user's machine
1/3 x 8 sec ~2.5 sec not very noticeable
Could be public in view of passphrase + secure delay Security
ENCRYPTED "LOCKED" PRIVATE KEY
publicly available
USER'S PUBLIC KEY
PUBLIC KEY / PRIVATE KEY PAIR GENERATOR
Once
Once
k
USER'S PRIVATE KEY
SYMMETRIC ENCRYPT.
Many
Many
very sensitive, never stored anywhere


XOR
INITIALIZATION VECTOR INPUT
(FIXED FOR FIRST CV PAIR, RESULT OF PREVIOUS CV PAIR FOR OTHERS)
BLOCK CIPHER
key
DELAYED, ENCRYPTED OUTPUT
(A,B) INPUT
KEY LOOKUP TABLE 2^M KEYS
USER SYSTEM INFO
N ITERATIONS
COUNTER MOD 2^M
M
Truly random best for preventing attackers from computing keys on the fly. (To avoid using memory)

1/2

Number of consonants	13
Number of vowels	5
Combinations in each CV pair	65
Pairs	8
Total Combinations	**318,644,812,890,625**
Base-2 Entropy	48 (bits)

Mean Input Times (experiment)

Touch typing (fast), hidden digits	4.70 (sec.)	=(5.3+ 4+ 5+ 5.1+ 4.1)/5
Tough typing (fast), digits shown	3.88	=(3.8+ 3.5+ 3.4+ 3.5+ 5.2)/5
Mouse, drag line through digits	9.32	= (9.7+ 9.2+ 9.8+ 8.9+ 9)/5
Mouse, click on digits	8.28	= (8+ 8.2+ 8.9+ 8+ 8.3)/5

Set total delay to minimum total input time (Keeps user from noticing the delay) 3.88 (sec.)

Software (equivalent machine) Attack Analysis

Total number of seconds for all delayed combinations (on equivalent machine)	1,236,341,874,015,620
Average number of years on equivalent machine (1/2 total)	19,602,072 ← Impossible with present machines
Effective lifetime of signing key (years)	20
Performance multiplier at end of life (Moore's law)	10,321
Total number of seconds for all delayed combinations (on future machine)	119,785,790,491
Number of future machines in network	1,000 ← Even this number would leave evidence of fraudulent activity on the part of the person forging signature with broken private key
Average number of years on future machine network (1/2 total)	**0.95**

Massively Parallel Hardware (FPGA, ASIC) Attack Analysis

Budget (current equivalent dollars)	1,000,000 ←
Cost per FPGA or ASIC (with NRE)	400
Number of available parallel branches in budget	2,500
Number of parallel branches operating simulateously	2,048
Performance multiplier of each branch over equivalent machine	100
Total performance multiplier over equivalent machine	204,800
Total number of seconds for all delayed combinations	6,036,825,557
Average number of years (1/2 total)	96 ← Impractical with present H/W, even w/ large budget
Effective lifetime of signing key (years)	20

↑ Can Include sunset date in ACI after which all sigs. are invalid

Z-9

2/2

Performance multiplier at end of life (Moore's law)	10,321
Total number of seconds for all delayed combinations (on future hardware system)	584,892
Average number of days on future machine (1/2 total)	**3.38**

But, here's where the key lookup helps protect against such attacks...

Random keys in key lookup table	8,192
Size of each key (in bytes)	16
Total memory for lookup table (bytes)	131,072
Total fast SRAM memory for all branches (bytes)	268,435,456
Total MB of fast SRAM memory	**262,144**
Cost per MB of SRAM (current equivalent dollars)	10
Total cost of SRAM	**2,621,440**

(See budget above.)

Should (must fit in 256K cache) to ensure top performance in user's machine. otherwise ratio between

$$\left[\frac{\text{Attacker Delay}}{\text{User Delay}}\right]$$

is reduced.

Lots of gates: $$

~ 8 x 2 gates per byte, or

~ 4.3 x 10^9 gates

or ~2M gates per branch

Expensive ASIC!

SECURE DELAY OF HASH OUTPUT

A secure delay according to various aspects of the present inventions can be applied in other areas than just passphrase security. For example, a hash value can be run through a secure delay to produce a smaller hash value that would be computationally infeasible to derive based on a birthday attack. In one conceived embodiment, a 160-bit hash value is repeatedly run through a secure delay for a predetermined number of iterations that, given a security selection, corresponds to an acceptable unit delay. (An example of an acceptable unit delay is 1 second.) At the end of each unit delay, a sub-hash is computed from the current output of the secure delay and displayed.

A person wishing to compare hash values can begin comparing a first group of digits corresponding to the first sub-hash after the first unit delay, and while the second unit delay is underway. When the person looks for the second group of digits to compare, the second unit delay (when optimally chosen) is already completed and the third unit delay is underway.

Thus, a securely delayed hash system according to various aspects of the inventions can provide a smaller hash value with the same security as the larger hash value from which it is derived. The loss in entropy in the smaller value is offset by the computational difficulty (from the secure delay) of obtaining the smaller value. An attacker wishing to find a larger hash value that produces the smaller hash value will need to run the secure delay, on average, N2/2 times with the secure delay computation for each iteration.

If T = delay time (on an equivalent processor as that of the legitimate user), then T2 = T*N2/2. If T = 1 second, and the required T2 = 1,000,000 CPU years, then the required N2 ~= 2^21, a much smaller value than, say, 2^160.

Since a hash value is not particularly sensitive, it can be sent freely over in secure networks. It is conceivable that an Internet site can be established for quickly computing smaller "sub-hashes" based on transmitted hash values through an open-source, standardized secure delay algorithm. However, it is more likely that the market will demand simplicity and standardization, and an average delay within the reach of the average desktop PC. (The remote-computation model may be useful for portable computers, though.)

# # #

Choices: $7 \times 5 = 35 = M$

Digits: $N = 8$

No duplicate digits

$\therefore$

$$X = M \cdot (M-1) \cdot (M-2) \cdot (M-3) \ldots (M-N-$$

$$X = \frac{M!}{(M-N-2)!} = \frac{35!}{25!}$$

$$X = 6.67 \times 10^{14}$$

$\log_2(X) = 49.2$ bits of entropy

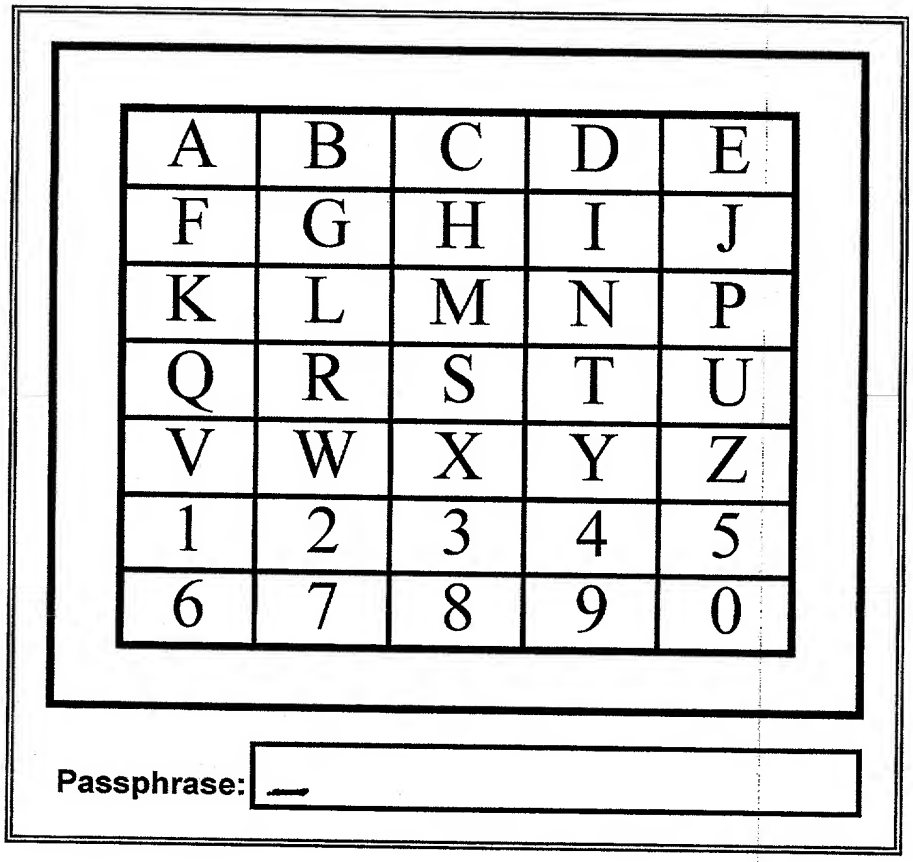
With 0.5 sec. delay between digits,
$d = 4$ sec

$$\frac{4X}{(3600 \times 24 \times 365)} = 84,496,818 \text{ yrs } (!)$$

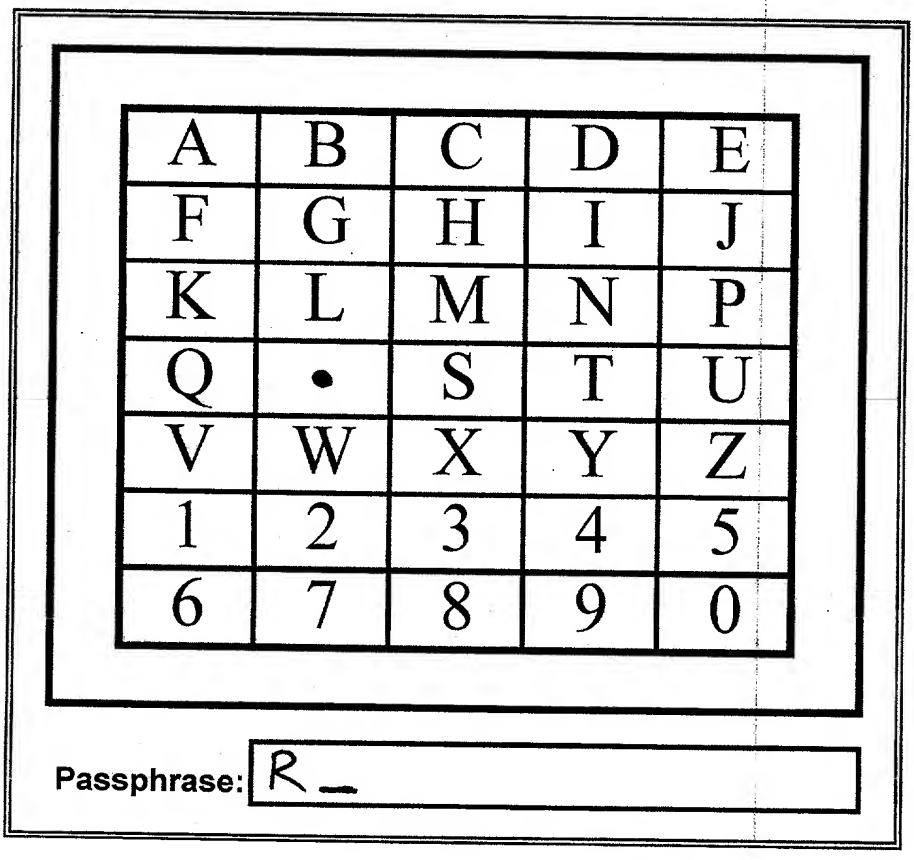
(sec/hr, hr/d, d/yr)

on comparable delay processor w/ 2x every @ 18 mos, in 20 yrs: 8,187 yrs (!)

AB-1



In a variation, entry system is built into keypad of USB-connected hardware delay processor



Already-selected digits can be illuminated

A	B	C	D	E
F	G	H	I	⠑
K	L	M	N	P
Q	⠊	S	T	U
V	W	X	Y	Z
1	2	3	4	5
6	7	8	9	0

Passphrase: RJ_

A	B	C	D	E
F	G	H	I	⠑
K	L	M	N	P
Q	⠊	S	T	U
V	W	X	Y	Z
1	⠘⠂	3	4	5
6	7	8	9	0

Passphrase: RJ2_

A	B	C	D	E
F	G	H	I	⠊
K	L	M	N	P
Q	⠏	S	T	U
V	W	X	Y	Z
1	⠁	3	4	5
6	⠂	8	9	0

Passphrase: RJ27 _

A	B	C	D	E
F	G	H	I	⠊
K	L	M	N	P
Q	⠏	S	T	U
V	W	⠭	Y	Z
1	⠁	3	4	5
6	⠂	8	9	0

Passphrase: RJ27 x_

AB-4

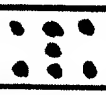
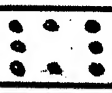
A	B	C	D	E
F	G	H	I	
K	L	M	N	P
		S	T	U
V	W		Y	Z
1		3	4	5
6		8	9	0

Passphrase: RJ27 XQ_

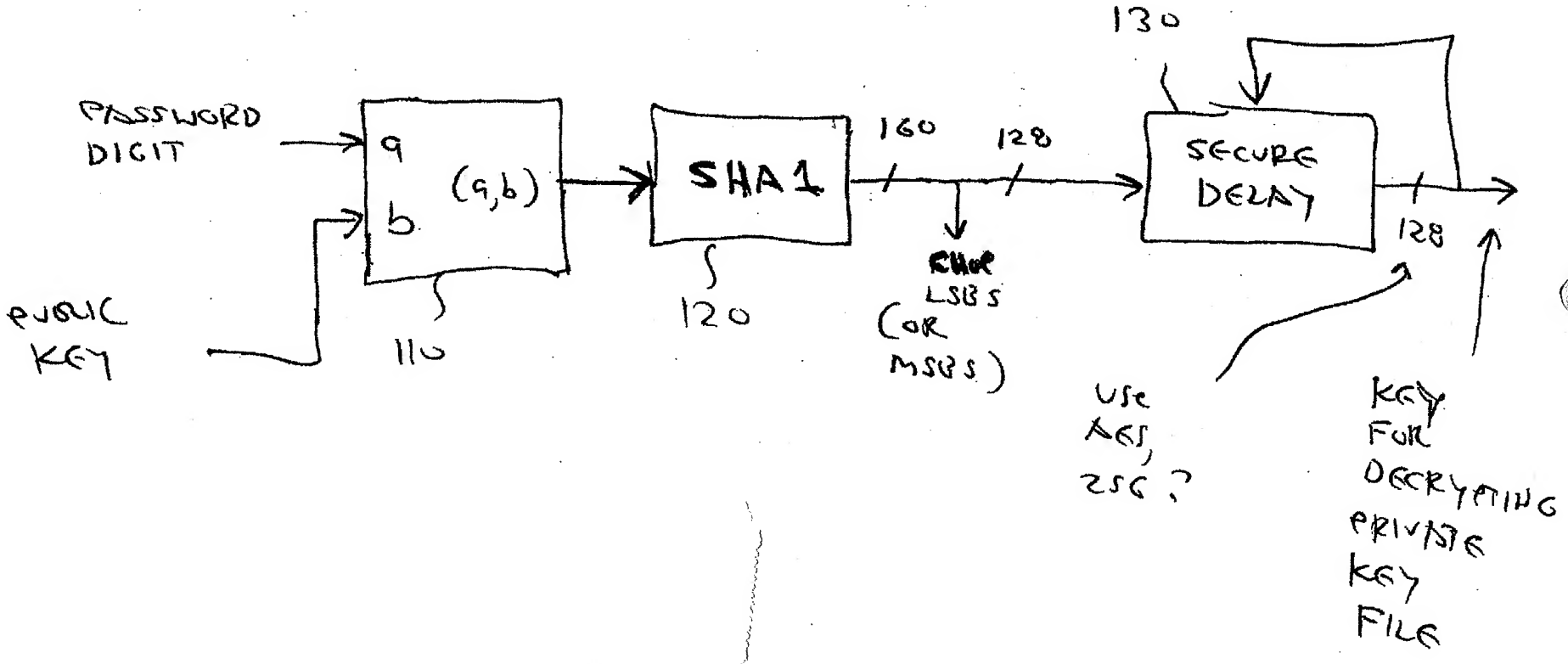
A	B	C	D	E
F	G		I	
K	L	M	N	P
		S	T	U
V	W		Y	Z
1		3	4	5
6		8	9	0

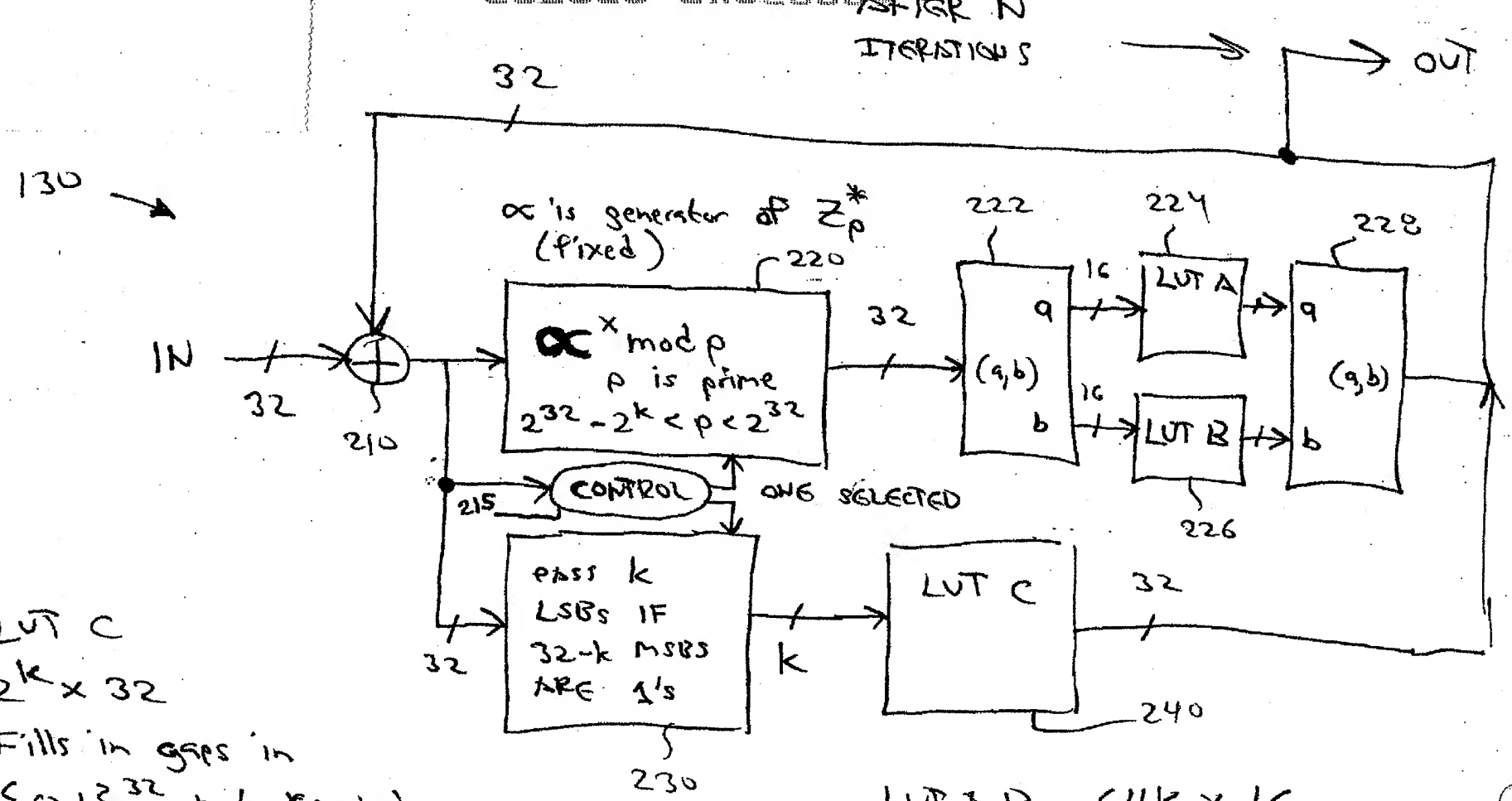
Passphrase: RJ27 XQH_

AB-5

A	B	C	D	E
F	G		I	
K	L	M	N	P
		S	T	U
V	W		Y	Z
1			4	5
6		8	9	0

Passphrase: RJ27 XQH3





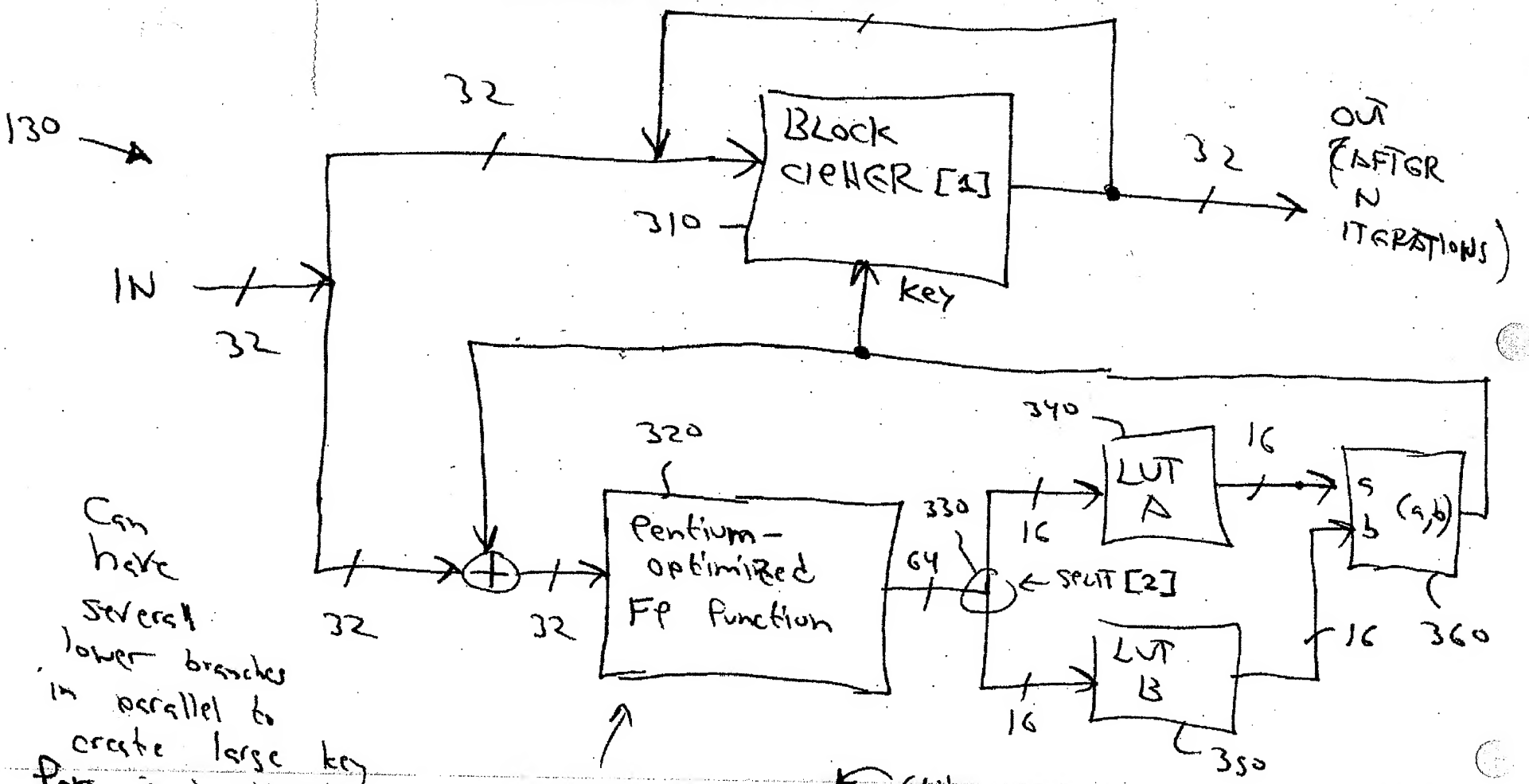
LUT C
$2^k \times 32$
Fills in gaps in $\{0,1\}^{32}$ not reached by LUTS (A,B) because $p < 2^{32}$, $\alpha^x \bmod p$ not selected for $x \geq 2^{32} - 2^k$.

ASSUMES
THERE IS A PRIVATE PRIME $< 2^{32}$ and WITHIN 2^k of 2^{32}
k small (4-5)

LUT A,B 64K x 16
Can be same random mappings of $\{0,1\}^{16}$ or different.

ONE EXEMPLARY EMBODIMENT OF 32-BIT SECURE DELAY

AC-2



[1] SHOULD BE FAST, SIMPLE SO FP FUNCTION DOMINATES PROCESSING (MAYBE JUST XOR?)

[2] SPLIT: SEND: BITS 48-63 TO LUT A
BITS 32-47 TO LUT B

ANOTHER EMBODIMENT OF 32-BIT SECURE DELAY

XC-3

VIRTUAL SIGNATURE PRINTER

According to various aspects of the present inventions, an electronic record (e.g., MS Word document, AUTOCAD drawing, etc.) can be signed by "printing" that record to a special "virtual signature printer." An example of a virtual printer is the "PDF Writer" printer driver that is installed with the ADOBE ACROBAT software. The inventive "virtual signature printer" provides the user with an intuitive, simple way of authenticating an electronic record.

The "virtual signature printer" system produces an output file (or multiple files, see below) that can be sent to a recipient for viewing, printing, and validation. The recipient can view or print the file (preferably, the file is "backward compatible" compatible with widely available viewing software) and, with special software, can validate the signature on the file.

A particularly advantageous way of signing an electronic record that has been "printed" this way is with embedded signatures. However, a "virtual signature printer" system can employ any suitable technique for signing an electronic record. The following are some examples of output of such a system:

- A PS or TIFF file (or, with suitable licensing if necessary, a PDF file) representing the document, accompanied by a detached PGP-compatible signature file.
- A ZIP file containing a PS, TIFF, or PDF file representing the document including a ZIP comment containing a Base-64 PGP-compatible signature.
- A PGP-signed file containing the document in a PS, TIFF, or PDF file.

When the signer wishes to electronically sign a document he or she can print the document to the virtual signature printer driver, using the print functionality of the software used to create the document. The printer driver creates a window in which the software requests the signer's authenticating information. The user can enter his or her passphrase, apply his or her fingerprint to a fingerprint scanner, insert a smart card, etc. The software then computes the digital signature for the document, based on the authenticating information or a private key unlocked by the authenticating information, and embeds the digital signature with an output file or includes the digital signature in a separate file.

# # #

EDWIN A. SUOMINEN

U.S. Patents
5,926,513; 5,937,341*;
6,052,748*; 6,069,913
Additional patents pending*

*Available for licensing

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Engineering Technology

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ed@eepatents.com

EXAMPLE

VIA EMAIL
Thomas E. Workman, Jr., Esq.
Law Offices of Thomas Workman
41 Harrison Street
Taunton, MA 02780

Re: Cryptography Technology

Dear Ted:

This brief disclosure is an example of a clear-signed document with a simulated digital signature (25x13 bits = 325 bits) that resides as a graphic within a signature block that is excluded from the signature calculation of the document. In an actual implementation, the user will place the signature block in the document and will then "print" the document to the digital signature printer driver. The driver will then create a graphic file such as a TIFF (multi-page if necessary), remove the graphics in the region where the signature block will go (setting that graphic data to a default blank value), compute signature for the entire document except the signature region, and place the signature data in the region as graphic-mode text.

The document's signature can be verified simply by opening it with a customized TIFF reader, which will detect the presence of the signature region and will validate the signature within it against the data of the document except the graphics within the signature block. An option can be provided to put the signature data on an entirely separate page of the document (e.g., after the last page), preferably with a facsimile copy of the signer's ACI. (In such embodiments, the ACI should have a blank space for the signature data of document signed with the ACI's signing key.)

An exemplary process is illustrated in the attached FIG. (AE-4), in which a signer creates a document (e.g., this letter) using whatever software he (or she) wishes to use (e.g., Microsoft Word 97). He then prints the document to the SelfCertify.com virtual TIFF printer (call it a "software signature machine"?). The printer driver software creates a TIFF file of the document and displays it in a viewer window. The user interface of the viewer window requests the following input from the signer:

1. Selection of a graphical region within the displayed document for application of the signer's digital signature "stamp." The user can specify the region by moving a dashed box around the screen. The dashed box can have left and right arrows within it for navigating to different pages of a multi-page document, and an "Sign Here" or "OK" button for applying the digital signature "stamp" at the

current location of the box. An unsigned signature page of this letter is attached showing what such a box might look like as it is moved around during the selection process.

2. Authentication from the signer to apply his digital signature to the document within the digital signature "stamp" at the selected location. As discussed in other disclosures, the authentication can be a securely delayed passphrase input.

When the signer has selected the location of the digital signature stamp and provided authentication for its creation, the printer driver software removes all graphic information from the selected location. It then computes the digital signature based on (1) the remaining data of the TIFF file (exclusive of the location of the stamp) and (2) the signer's private key.

Advantageously, the signed document is in a conventional format (e.g., multi-page TIFF) that can be read by any conventional viewer is signature authentication is not needed. When signature authentication is needed, a document can be viewed using the SelfCertify.com TIFF viewer (which may be distributed freely to encourage use of SelfCertify.com's digital signature services).

To verify a digital signature, the viewer searches the graphical rendering of the signed document for the distinctive graphical outline of the signature block. Distinctive features of the graphical outline can include (1) a distinctive color such as maroon, (2) a distinctive line shape such as double parallel lines, and (3) a distinctive line weight such as 3.2 points (not an integer or x.5 fraction). All of these distinctive properties are present in the simulated signature block for this document. In addition, the signature block can have a predetermined size to further identify it.

Once the viewer software has identified the exact location of the signature block, it removes all of the graphic data of the signature block (up to and including the border) and sets it to the default blank value used during signature calculation. It performs an optical character recognition of the signature data within the block to obtain the value of the digital signature. According to advantageous aspects of the invention disclosed elsewhere, the digital signature can be applied to a secure delay to obtain another, larger digital signature value. The digital signature value (as shown or as expanded through a secure delay) is then validated against the modified TIFF representation of the document.

Very truly yours,

-SelfCertify.com Digital Signature-
1410-2708-9007-5781-6673
7701-3376-8745-3789-9100
7440-2788-7101-4089-2409
5420-2178-9001-7680-1477
7731-3376-2645-3789-9105

Edwin A. Suominen

NOTE: Signature data can be in the form of a bar code (1D or 2D). If license to PDF format could be obtained, could include signature characters as data (not image pixels) and search would be very simple search for key tag characters in file.

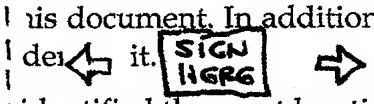
current location of the box. An unsigned signature page of this letter is attached showing what such a box might look like as it is moved around during the selection process.

2. Authentication from the signer to apply his digital signature to the document within the digital signature "stamp" at the selected location. As discussed in other disclosures, the authentication can be a securely delayed passphrase input.

When the signer has selected the location of the digital signature stamp and provided authentication for its creation, the printer driver software removes all graphic information from the selected location. It then computes the digital signature based on (1) the remaining data of the TIFF file (exclusive of the location of the stamp) and (2) the signer's private key.

Advantageously, the signed document is in a conventional format (e.g., multi-page TIFF) that can be read by any conventional viewer is signature authentication is not needed. When signature authentication is needed, a document can be viewed using the SelfCertify.com TIFF viewer (which may be distributed freely to encourage use of SelfCertify.com's digital signature services).

To verify a digital signature, the viewer searches the graphical rendering of the signed document for the distinctive graphical outline of the signature block. Distinctive features of the graphical outline can include (1) a distinctive color such as maroon, (2) a distinctive line shape such as double parallel lines, and (3) a distinctive line weight such as 3.2 points (not an integer or x.5 fr[illegible] :operties are present in the simulated signature block for [illegible]is document. In addition, t[illegible] signature block can have a predetermined size to furthe[illegible] de[illegible] it.

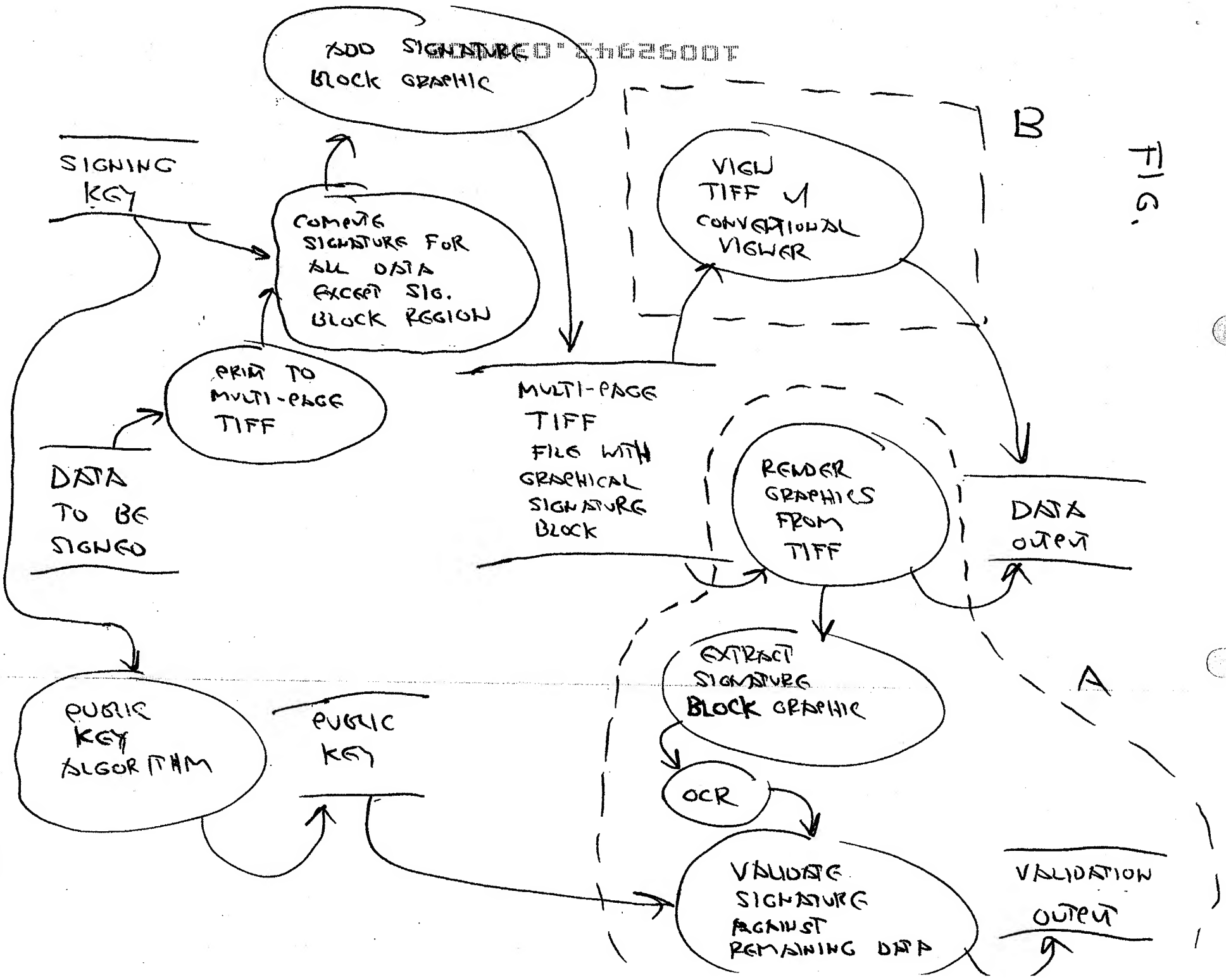


Once the viewer software ha[illegible] identified the exact location [illegible] the signature block, it removes all of the graphic data of [illegible] ncluding the border) and sets it to the default blank value used during signature calculation. It performs an optical character recognition of the signature data within the block to obtain the value of the digital signature. According to advantageous aspects of the invention disclosed elsewhere, the digital signature can be applied to a secure delay to obtain another, larger digital signature value. The digital signature value (as shown or as expanded through a secure delay) is then validated against the modified TIFF representation of the document.

Very truly yours,

Edwin A. Suominen

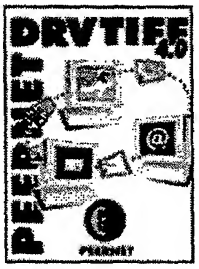
AE-3


ADD SIGNATURE BLOCK GRAPHIC
SIGNING KEY
COMPUTE SIGNATURE FOR ALL DATA EXCEPT SIG. BLOCK REGION
PRINT TO MULTI-PAGE TIFF
DATA TO BE SIGNED
MULTI-PAGE TIFF FILE WITH GRAPHICAL SIGNATURE BLOCK
VIEW TIFF W/ CONVENTIONAL VIEWER
B
RENDER GRAPHICS FROM TIFF
DATA OUTPUT
EXTRACT SIGNATURE BLOCK GRAPHIC
A
OCR
PUBLIC KEY ALGORITHM
PUBLIC KEY
VALIDATE SIGNATURE AGAINST REMAINING DATA
VALIDATION OUTPUT
FIG. 4

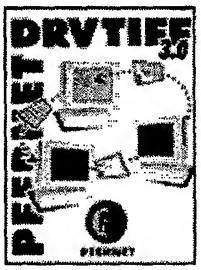
ALTERNATIVE: Conventional TIFF + detached pgp signature (e.g. in Zip file.

PEERNET.DRV TIFF Driver

Selecting a Driver | Supported Platforms | Common Features | Support | Online Solutions



4.0 online MANUAL
14 day TRIAL 4.0
3.0 online MANUAL
14 day TRIAL 3.0



PEERNET

Enterprise Labeling Suite

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Free Trials

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PEERNET.DRV TIFF Driver converts any document capable of being printed by a Windows application into high quality serialized or multi-page TIFF images. It is ideal for imaging or archiving applications. It's also a handy file-generation tool for cross-platform article distribution.

TIFF conversion is as fast as printing. Document scanning is obsolete. Paper waste is a thing of the past.

Features of PEERNET.DRV TIFF Driver

Append Mode (Version 4 only)
Build a multi-page file gradually by appending pages to an existing file whenever the need arises.

Color Correction
Lets you adjust the image's appearance to compensate for monitor non-linearity on Windows NT 4.0 and Windows 2000.

Color Modes
Color or "Black and White only".

Color Reduction
Turn automatic color reduction on or off, to suit your needs. Automatic color reduction reduces your image to its fewest number of colors without affecting picture quality. In addition, automatic color reduction selects the optimal output format for serialized files or the optimal output page format for multi-page files.

Compression
Turn compression on or off as needed.

Output Formats
TIFF True Color Uncompressed or Compressed Packbits
TIFF Monochrome Uncompressed or Compressed CCITT Group 4
TIFF 256 Color Uncompressed or Compressed Packbits

Paper Size
Converts custom documents up to 18.03 x 18.03 inches, or 458 x 458 millimeters. Supports most paper sizes.

Resolutions
100, 200, and 300 DPI resolutions.

Can use existing technology to print signed documents according to various aspects of the present inventions.

AE-5

ZDNet DOWNLOADS

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Another example of existing TIFF generation technology that may be adapted for use according to various aspects of the present invention.

EZ-Printer for Windows NT

10-26-99

Print to image file from any application

ZDNet Rating ★★★★ **User Rating** NR

Be One of the First to Rate This Title!

Company: Suntop
Version:
Size: 774.95 KB
Downloads: 485

Download Now

Get GO!ZILLA And Download Better

Add to Basket
Look in Basket

• View file contents

Requirements: Windows NT 4.0

Purchase Information: Demo: $49-$59 for retail version.

EZ-Printer for Windows NT makes it possible to print from any application to an image file. It installs itself as a native device in the printer control panel. Users can simply choose Print and select the EZ-Printer printer from the list of available printers. Output is in black and white (the author has a color version, too) and is automatically saved in the file format of choice (DIB BMP, GIF, TIFF, or PNG). The images are automatically sequentially numbered. The included viewer will let users see the results immediately. This demo version of the driver includes a banner in the middle of the image. Virtually no documentation is included, so users will need to visit the authors' Website to get information or support. Output also seems to be limited in resolution, with no apparent way to control the dpi. Still, EZ-Printer could be particularly useful for Websites, writing documentation, presentations, and many other applications.
Reviewed on Oct 23 1999.

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PRINT TO SIGNED TIFF FILE

Appendix AF

According to various aspects of the present inventions, a document can be signed by printing the document to a TIFF file using a virtual printer driver, e.g. provided by a service such as "SelfCertify.com." (Note that SelfCertify.com is not an operating business entity, though the applicant has registered the domain name, and has not offered any product or service for sale as of the filing date of the present provisional application.) The TIFF file is created as it normally would be except that includes a signature block within a suitable field.

In one embodiment, the signature block is included within the TIFF "ImageDescription" field, the ASCII contents of which are excluded from the signature calculation of the file. (See AF-3.) In an actual implementation, the user "prints" the document to the digital signature printer driver. The driver will creates the TIFF (multi-page if necessary), with blank characters in the fixed-length "ImageDescription" field where the signature data is intended to reside (setting that data to a default blank ASCII value), compute signature for the entire document with the default blank value in the "ImageDescription" field, and place the signature data in the "ImageDescription" field as ASCII.

The document's signature can be verified simply by opening it with a customized TIFF reader, which extracts the signature data from the "ImageDescription" field and validate it against the data of the document with default blank values substituted in the "ImageDescription" field.

An exemplary process is illustrated in the attached FIG. (AF-4), in which a signer creates a document using whatever software he (or she) wishes to use (e.g., Microsoft Word 97). He or she then prints the document to the SelfCertify.com virtual TIFF printer. The printer driver software creates a TIFF file of the document and displays it in a viewer window. The user interface of the viewer window requests authentication from the signer to apply his digital signature to the document. The authentication can be a securely delayed passphrase input according to various aspects of the present inventions.

Advantageously, the signed document is in a conventional format (e.g., multi-page TIFF) that can be read by any conventional viewer is signature authentication is not needed. When signature authentication is needed, a document can be viewed using the SelfCertify.com TIFF viewer (which may be distributed freely to encourage use of SelfCertify.com's digital signature services).

The signer's public key can be included in the signature data along with the digital signature and the name of the signer. A facsimile copy of the key ACI (see, e.g., Appendix A) can be appended to the end of the TIFF file after the document pages.

Having both of these items present in the TIFF file permits a verifier to quickly authenticate the signed document. If he or she is satisfied with the integrity of the TIFF reader/verification software he or she has obtained (e.g., from a secure, trusted web page of SelfCertify.com), and if he or she is satisfied that the facsimile copy of the ACI with its security background digits is not a forgery, he will have everything he needs to validate the signature of a person who has not made any prior digitial assigning arrangements with him.

Signing and verification software according to various aspects of the present inventions can be integrated with the GPG ("GNU Privacy Guard") software, which can be freely distributed and modified under the GNU Public License. The signing and verification software can call the GPG software with parameter passing. The inventor of signing and verification software can thus include, generally, a slightly modified TIFF printer driver and TIFF reader that calls the GPG software for all digital signature functionality. All three pieces of software can be released in a single compact package.

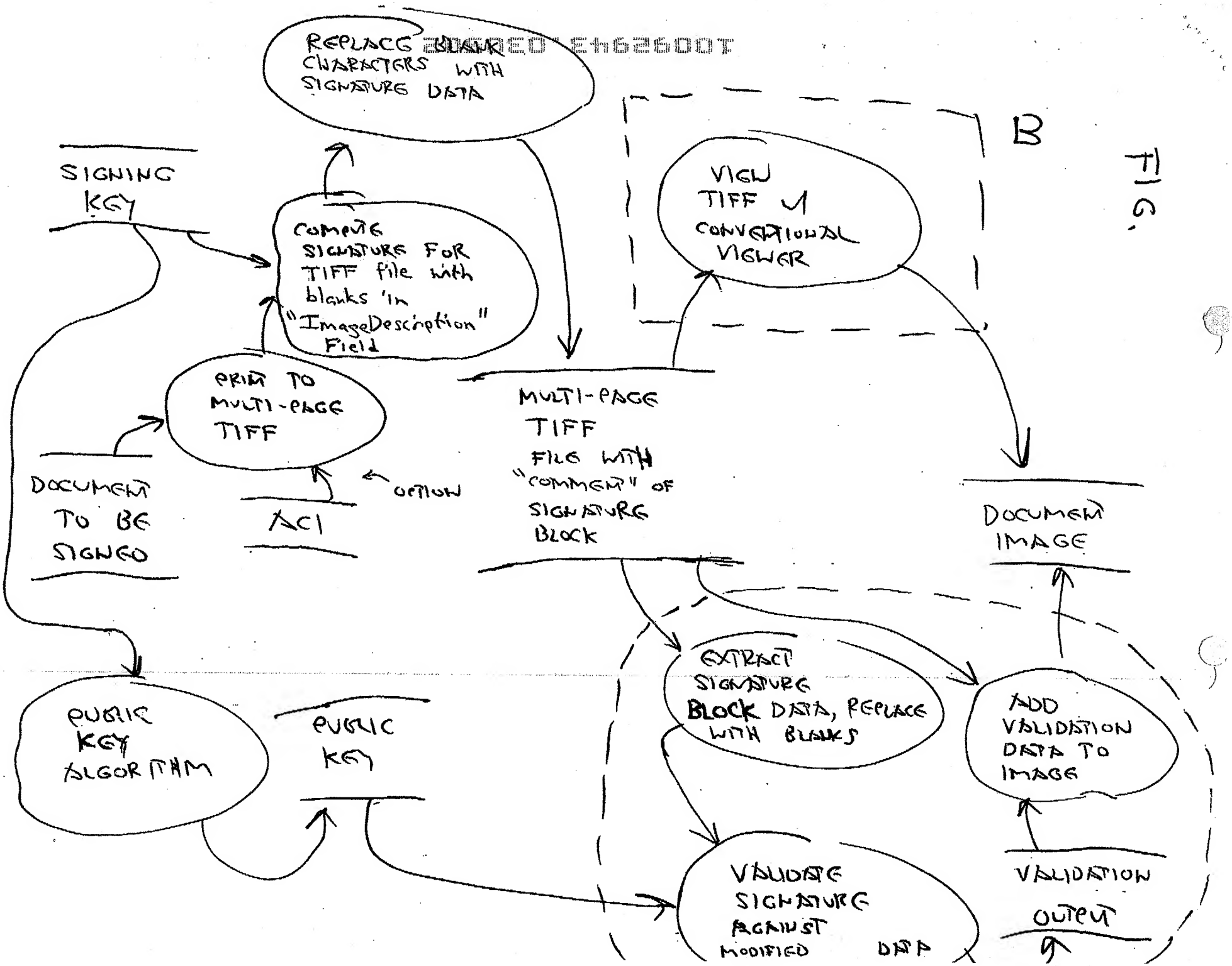
The TIFF reader software can provide the option to output the original TIFF file (without signature data in the "ImageDescription" field) along with a PGP-compatible detached signature to the file and an ASCII file with the signer's public key. This permits less trusting users to verify integrity of the signature and signing key (comparing PGP's SHA-1 "fingerprint" to what's shown on the ACI) with their own copy of PGP or GPG.

###

Sample "ImageDescription" Tiff Field

Tag = 270
Type = ASCII
Count = Fixed number of characters in signature block, plus 1 for NUL at end.

Value = 1728
"-- SELFCERTIFY.COM SIGNATURE DATA -- ↵
Signed by: Edwin A. Suominen ↵
mQGiBDk8ZlMRBADZZ0behMne0qwL7mu7fa/KfbPx2wLtMSihh3IitOo6o6e/twYQ ↵
3Z27YlIlu9uvhIkdsBrQ7b+N0paKyJAu691eE5gzP8VEdzfLJtCQDXvdO9+H57Er ↵
PGicVujuGGIPxvzA7QuyxNxDzndKtFIGO60zn452pWrg/77iA+Ne0CYCuQCg/9I3 ↵
6bNa0vfxxUV3CS+/PDo9VpED/jwquHOyJQYOi0jZNdaT9ZN8mzRQlPgfyGHuNBpp ↵
yoPRhck0nMe1BxNG83M2v243M0DUmabCPuGqqOtYKe5YLqAw/iM5IWwp3EctEHGU ↵
48r8gC7rYKRHOLosyBfx6/uQkpGiNeM4TAI9QpqVzsbHvF1H5LSauLbHOSAwTUoM ↵
e/+uBACTQjV15XA+MPwaIx1vZ3lD2iEX/XFLOedxA5XzcN9uVlet+Chxd7xJn66x ↵
w0xzdVLvb/kCdcNY8idJkJVqAUx249S+PymCQFR+sX0pxXCVky4JgtLDToX0wW1G ↵
6C0kPzUR1AH9jdAUaPc+7SC1TdixmOPsLR1l+5PzUUVhNj6b8bQgRUFTIEp1bmUg ↵
MjAwMCA8ZWRAZWVwYXRlbnRzLmNvbT6JAFQEEBECABQFAjk8Zl1MFCQAiRwAECwMC ↵
AQIZAQAKCRCRZ8BksUXUY2Y3AKCG99iXRgxGmOssyOC0Lwm/U0yECACfW6R9rI2f ↵
G+UeNOWE/b2TJDt49La5Ag0EOTxmUxAIAPZCV7cIfwgXcqK61q1C8wXo+VMROU+2 ↵
8W65Szgg2gGnVqMU6Y9AVfPQB8bLQ6mUrfdMZIZJ+AyDvWXpF9Sh01D49Vlf3HZS ↵
Tz09jdvOmeFXklnN/biudE/F/Ha8g8VHMGHOfM1m/xX5u/2RXscBqtNbno2gpXI6 ↵
1Brwv0YAWCvl9Ij9WE5J280gtJ3kkQc2azNsOA1FHQ98iLMcfFstjvbzySPAQ/Cl ↵
WxiNjrtVjLhdONM0/XwXV0OjHRhs3jMhLLUq/zzhsSlAGBGNfISnCnLWhsQDGcgH ↵
KXrKlQzZlp+r0ApQmwJG0wg9ZqRdQZ+cfL2JSyIZJrqrol7DVekyCzsAAgIH/0V8 ↵
DY5pj51RDGsakRhMebL9Ob7v9GsbZN6PfTgO2upuCi6WUyazabw4J4ZFc7vtpo8x ↵
FQOkCofOLmisNim7r0PyWrW0SgHLbcXwMMUUb1h/QbggH0WtkkJTzxgNGL+MLJZa ↵
ND4R0gle03PQep4SZgA6/x9OUGWStmzWEt3jk/VdnImS5gDJmNHmCX7+ZaCxROiL ↵
zO3oDmzIRpVYk3+tnekDVhhrDwX5lQ1zUoCg43hAmA1Q1/KNFBw/qiol0EvLyJby ↵
hUzhGqdzd/MJkNHXviOoJyuOnQH+O8lEME5S2Ej19epf4Rfuf9rn8uR7tl3YEraD ↵
wqwO4VIcd5n+6F3199GJAEwEGBECAAwFAjk8ZlMFCQAiRwAACgkQkWfAZLFF1GMG ↵
KwCZARTQgJDOM40GBp0OJwP1escVP/gAoPIJb/gmbNpbeQmG9UobWiT8PKl1 ↵
=zFrB ↵
↵
Signature: ↵
iQA/AwUAOhs1jKmKuMvNCWDGEQJ4TACeJpwTCOzNvxKhZVYag17lBEuhKEMAnjtT ↵
SivKAZgC21P/pMrro2HgTfJo ↵
=Adug<NUL>"


REPLACE BLANKED CHARACTERS WITH SIGNATURE DATA
SIGNING KEY
COMPUTE SIGNATURE FOR TIFF file with blanks in "ImageDescription" Field
PRIOR TO MULTI-PAGE TIFF
DOCUMENT TO BE SIGNED
ACI
OPTION
MULTI-PAGE TIFF FILE WITH "COMMENT" OF SIGNATURE BLOCK
VIEW TIFF W/ CONVENTIONAL VIEWER
B
DOCUMENT IMAGE
PUBLIC KEY ALGORITHM
PUBLIC KEY
EXTRACT SIGNATURE BLOCK DATA, REPLACE WITH BLANKS
ADD VALIDATION DATA TO IMAGE
VALIDATE SIGNATURE AGAINST MODIFIED DATA
VALIDATION OUTPUT

IMPROVEMENT TO PGP

By Edwin A. Suominen

Allowing keys of keyring to exist at different locations

I would like to follow the advice of some encryption experts to keep my private key (or at least a signing key) on removable media (e.g., a floppy disk) for added security. However, the only way to do this now is to have my entire PGP keyring on that floppy disk. If the key I'm truly worried about is to be Secure, I won't want to keep that floppy disk where it is easily accessible. Instead, I will want to keep my public keys and my encryption key on the hard disk and just my signing private keying on a floppy disk.

According to this proposed improvement, PGPkeys allows users to specify the file location of private keys on their keyring. Everything works normally except when an "indirect" private key is to be used, e.g., to apply an especially important PGP signature to a contract. When that happens, the PGP software looks for the file containing the indirect key. If the file is located on removable media that is not in the drive, the software prompts the user to insert the removable media containing the indirect key. If the file is located on a PGP disk or some other type of encrypted volume, the software could simply indicate that the volume is not present and invite the user to mount it.

After use of the PGP key is complete, the software can advise the user that it is no longer needed and that he or she can unmount the volume or put the removable media back in its secure location.

To:
From: Ed Suominen <ed@eepatents.com>
Subject:
Cc:
Bcc: ed@eepatents.com
Attached:

LAW OFFICES OF LOUIS J. HOFFMAN, P.C.
14614 North Kierland Boulevard, Suite 300 * Scottsdale, Arizona 85254
Telephone: (480) 948-3295 * Facsimile: (480) 948-3387

Edwin A. Suominen * Admitted to practice in patent matters before the U.S. Patent Office only
Web Site: http://eepatents.com * PGP Public Key: http://eepatents.com/key

12/30/2005 FRI 15:39 [ ]

```
-----BEGIN PGP SIGNED MESSAGE-----
Hash: SHA1
```

This is an example of a message that has been signed with **preserved formatting** and an *unobtrusive* digital signature around clear-signed text, according to various aspects of the present inventions.

```
-----BEGIN PGP SIGNATURE-----
Version: PGP Personal Privacy 6.5.8

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qtttmsePjh/WqGafymg/hVMs
=q4Oy
-----END PGP SIGNATURE-----
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AH-1

CRYPTOGRAPHIC DOCUMENT DESTRUCTION

BACKGROUND AND SUMMARY

In private confidential conversation, two people can have a conversation without leaving any record of their conversation. With written or electronic communications, however, there is some record of what was said. That record can be difficult to eliminate.

Paper communications such as letters can be shredded if both sender and recipient agree that they will destroy their copies. Electronic communications (e-mail) are more difficult to eliminate because backup copies can be made and automatically archived onto other locations. It is sometimes surprising that backup copies are available during discovery of communications that would be embarrassing. Accordingly, there is a need for a system of destroying electronic communications or records when the sender and recipient of the communications agree to do so.

A system according to various aspects of the invention includes: an encryption subsystem; and a decryption subsystem, the decryption subsystem using a temporary key that can be disposed of to make encrypted communications unreadable.

DETAILED DESCRIPTION

An encryption key allows an authorized person to decrypt encrypted communications. For example, an encryption key can be a passphrase, or use a passphrase, known only to a person authorized to decrypt communications. According to various aspects of the present inventions, the decryption key can be destroyed. A passphrase for such a decryption key is preferably forgettable, for example a random alphanumeric string of sufficient length to be secure. Advantageously, the alphanumeric string can be used as a passphrase to open communications or records when is desired to do so and then destroyed and forgotten about when it is no longer desired for such communications or records to ever be decrypted again.

Operation of one embodiment includes (1) writing an electronic mail message to a person; (2) encrypting the communications using an agreed-upon passphrase, preferably an alphanumeric random digit string, for example 12 digits in length; (3) sending the encrypted message to the recipient; (4) having the recipient type in the passphrase to open the encrypted communications; and then after a predetermined or agreed-upon period of time, (5) having both parties destroy the passphrase (throwing away a Post-It note upon which the passphrase is written) so that neither the sender nor the recipient can ever decrypt the communication again. Preferably, the passphrase is used only for a short length of time or limited number of times so that it is impossible

for either party to remember it. The more random and arbitrary cryptic the passphrase is, the more difficult it will be to ever remember.

Systems according to various aspects of the invention can be useful in the legal profession where sometimes legal professionals are called upon to testify about matters that were assumed to be privileged but the court determines that they are not for whatever reason, as happens in patent practice sometimes. If an attorney or agent has communicated with his client using this system, and the client agrees to destroy the passphrase after the matter is complete, and the device communicated by the attorney or agent is no longer relevant or needed and has been acted upon completely, then it is impossible for any court or any party to discover what to parties discussed.

Even if a backup copy of an electronic mail message is found, a court can authorize a cryptananalysis of the message, but if it is encrypted using PGP strong encryption, it would be very difficult, effectively impossible, for the opposing party to figure out what the message said.

Embodiments can be employed in other types of communications that are encoded in digital form so that they can be encrypted. Even handwritten notes can be scanned into digital form and encrypted.

Voice messages can be digitized and compressed, entire paper files can be archived by scanning and digitizing, and then encrypting into a single encrypted archive file with a temporary key that can be disposed of after a predetermined time, which can be set by policy, for example one year.

According to another aspect of the invention, the keys need not be remembered or typed in by a human operator at all. According to this aspect, the key is an actual hardware device that transmits decrypting authorization indicia for a predetermined or agreed-upon period of time and then is incapable of doing so after that. An example of such a key is placed between a conventional PC keyboard and a PC. The device includes circuitry for reading a decryption key code or indicia from another device such as a card having barcodes printed on it or a disposable integrated circuit, which can be made in the form of a key.

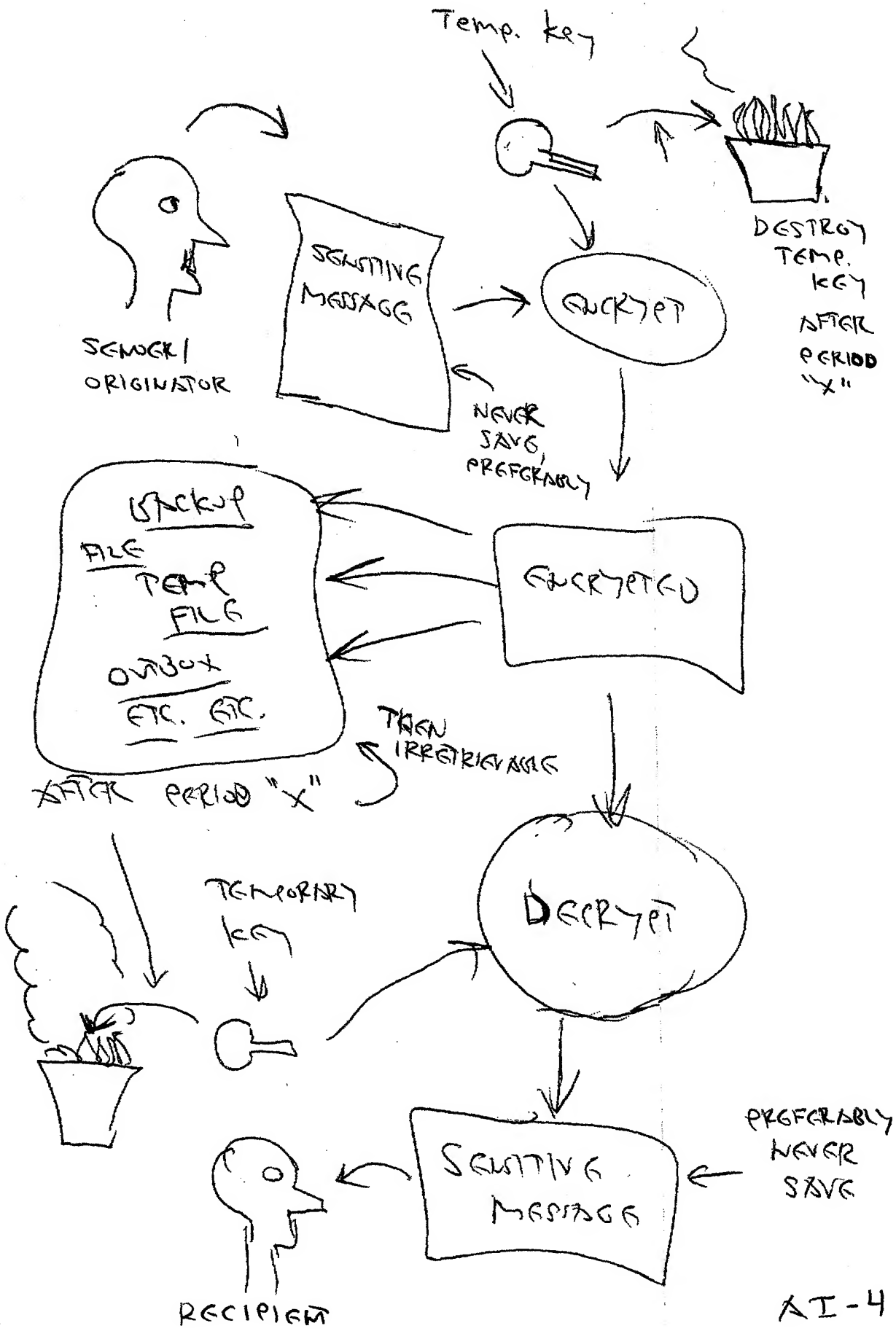
The device can be sold with a number of keys that can be used and disposed of by the user. For example, if the device is sold with twelve keys with refills of 12 additional keys available by ordering, the user can encrypted archive records every month with a different key and cost a new (different) key away every month. The user may wish to keep the records on file for a period of several months in which case the user will begin using a key one month and then put the key into storage for a couple of months and then toss the key away, destroying it irretrievably after that period of time.

An embodiment using printed cards is less expensive and the keys can be disposed of more cheaply but it is more prone to unauthorized or inadvertent duplication, in which case the whole purpose of the system might be defeated. The user of such a system needs to take precautions that the keys are never duplicated.

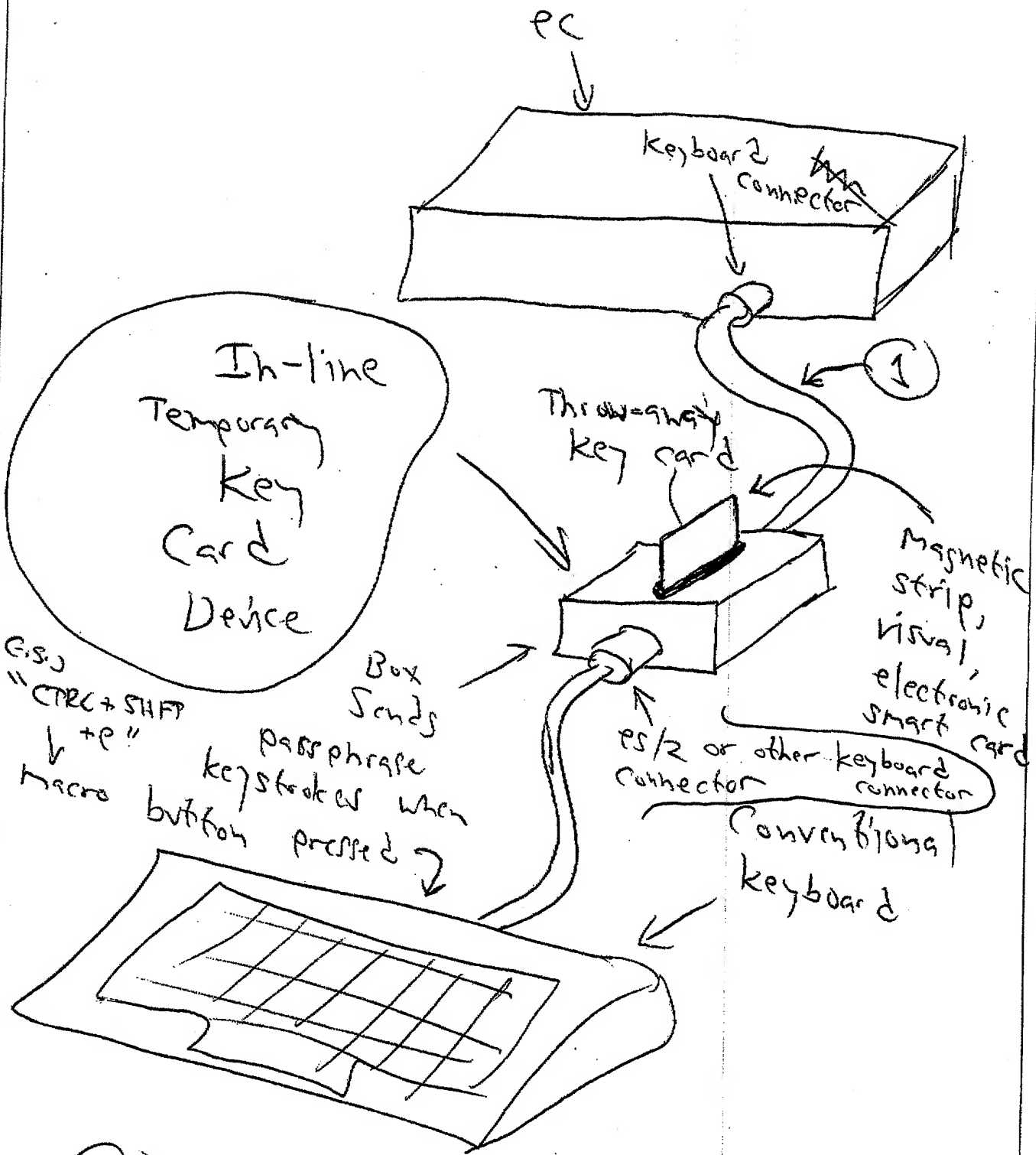
A database of which files correspond to which temporary keys can be created according to aspects of the invention so that an administrator can look over the list of keys about to expire and ask the persons involved with the effective files whether or not they need information from the files before they are destroyed. Paper documents can be shredded at the same time the keys are destroyed. If the key for a paper file that has a corresponding electronic file is a card, the card can be kept with the paper file and both can be destroyed simultaneously.

The key can be distributed from a sender of information to a recipient who is only authorized to access the information for a temporary period of time, for example one or two days. The sender of the information, or provider, can demand the key back after that period of time. In such a system, the key needs to be difficult to duplicate, for example an integrated circuit in the shape of a key. A forgettable password would not work for such a system because the user could write it down without telling the sender, but the forgettable password system works well when both parties, or all parties involved, are in cooperation and consent to destroy the information and the forgettable password.

# # #


Temp. key
DESTROY TEMP. KEY AFTER PERIOD "X"
SENDER/ ORIGINATOR
SENSITIVE MESSAGE
ENCRYPT
NEVER SAVE, PREFERABLY
BACKUP FILE
TEMP FILE
OUTBOX
ETC. ETC.
ENCRYPTED
THEN IRRETRIEVABLE
AFTER PERIOD "X"
TEMPORARY KEY
DECRYPT
PREFERABLY NEVER SAVE
SENSITIVE MESSAGE
RECIPIENT

AI-4



(1) Can be tamper alarmed, must send keystrokes if tap is detected

pass-phrase ("key")

preferably w/ secure delay procession in box here and AJ-1

also (can be used with permanent key systems)

Physical "private key" for conventional crypto. systems (e.g., PGP, hushmail)

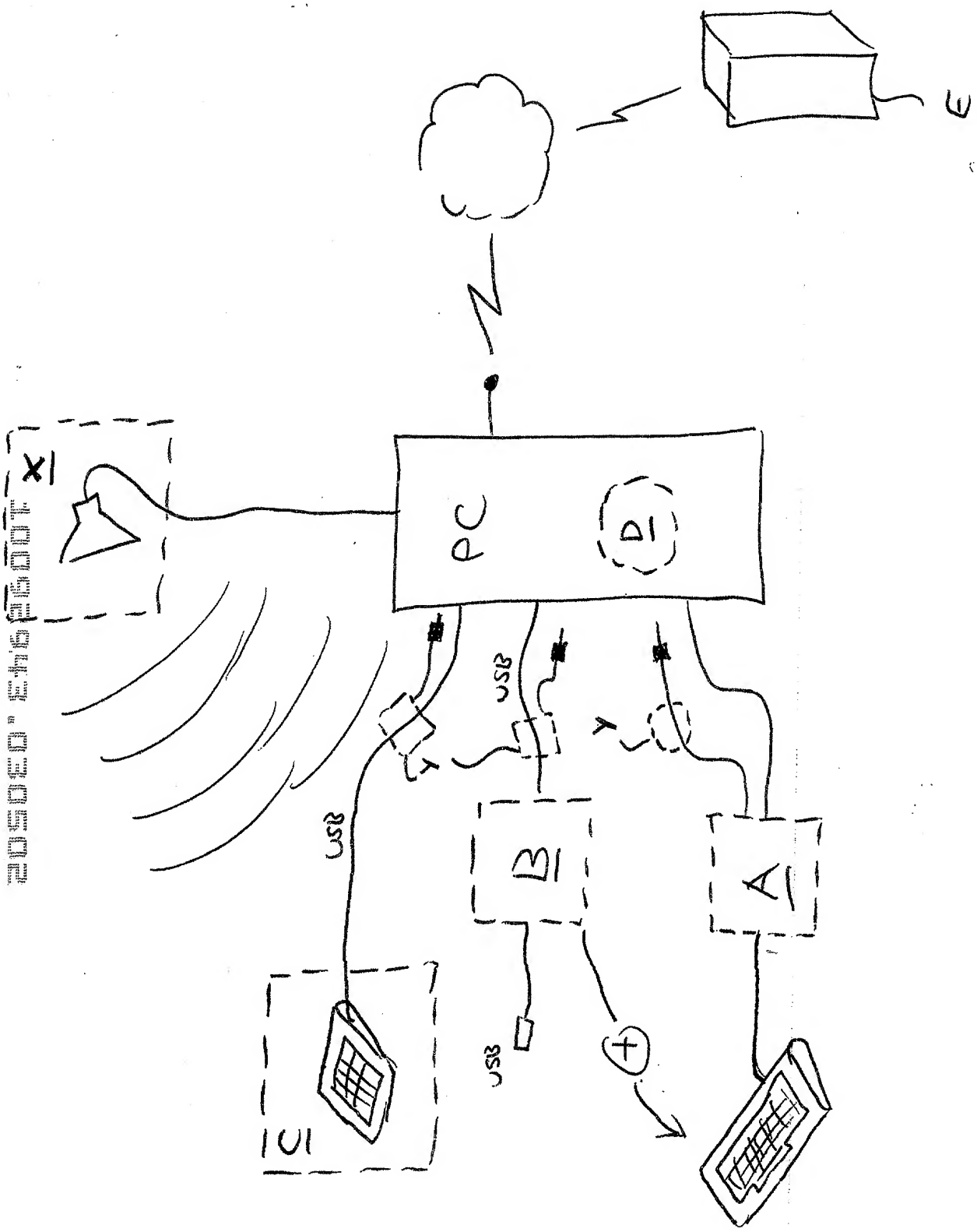
or magnetic card

In-line box, pass phrase stored on key, sent when key combo typed on conventional keyboard

*

or could hang off port, e.g., USB (but then raises question of shifting ports)

* or fingerprint (or other biometric) sensor w/ patterns as "passphrase" or "key"


E
X1
PC
S1
USB
USB
USB
B1
A1
U1

CLOCK JITTER RANDOM BIT GENERATOR

This invention uses existing hardware in a standard desktop PC with some very compact software to efficiently generate truly random numbers. Tell that introductory sentence to any cryptographer and they will either (1) beg you tell them how, or (2) dismiss you as an ignorant crackpot and lead you to various references talking about detecting hard disk read latencies, digitizing thermal noise, counting bubbles in fish tanks or lava lamps for generating true random numbers. (Pentium III chips have a hardware random number generator embedded in them, but I'm not sure that the AMD chips do. In any event, this invention provides random numbers from hardware that exists in all PCs today, and the need is still out there for a simple software solution.) I think if they were to read the following paragraphs they would scratch their head and say "Now why didn't I think of that?"

A method of generating a random binary bit according to various aspects of this invention includes: (a) clocking a CPU using a phase-locked loop; (b) beginning a count of CPU clock cycles upon a first transition of an independent clock signal (e.g., the PC's real-time clock, which generates interrupts on IRQ08 under control of a 32.768 kHz crystal); (c) recording the number of CPU clock cycles counted upon a second transition of the independent clock signal; and (d) extracting the least significant bit of the count to serve as a random binary bit.

Modern microprocessors use high-frequency clocks that are generated using a phase-locked loop (PLL). The PLL multiplies a lower-frequency crystal oscillator clock signal to generate microprocessor clock, as well as other clock signals used in the system. The Intel "CK98 Clock Synthesizer/Driver" is an example of a chip containing a PLL for this purpose.

PLL's lock the output frequency of a relatively unstable voltage-controlled oscillator to the more stable reference frequency of a crystal oscillator. The locking is performed using a feedback loop that has particular characteristics. Because phase locking cannot be perfect, there is always some "phase noise" or jitter on the PLL output. This phase noise or jitter causes the frequency of the PLL output signal to randomly vary within a Gaussian distribution. CPU clock generators are not designed to have very low phase noise because the exact clock frequency of the CPU is not particularly important. (That is in contrast to the type of PLL design I'm familiar with, for cellphone receivers, where the phase noise needs to drop off within a few hundred Hz of the carrier.) The PLL of the CK98 chip, for example, specifies that the "ideal closed loop jitter bandwidth" be attenuated 20 DB at 500 kHz, and a graph in the chip's data sheet shows the jitter (phase noise) spectrum

extending out to 50 kHz without significant attenuation.

With these levels of jitter, the number of CPU clock cycles in any given time interval can be expected to have significant variance. The high frequency components of the phase noise will integrate over long intervals and cancel each other out to some extent, but the number of CPU clock cycles over one predetermined time interval will be different (in a random fashion) from the number of CPU clock cycles over another predetermined time interval of the same length.

As a time interval gets longer, the number of clock cycles over the interval increases and the possibility of an integer difference between the number of clock cycles in two separate intervals of the same length increases proportionately. However, longer intervals permit high frequency variations in instantaneous frequency to cancel each other out.

The probability of any given frequency measured over a given time interval is defined as a Gaussian function of the frequency offset from the correct PLL frequency. Thus the probability of any given offset frequency measured over a larger time interval can be expected to be smaller than the probability of that frequency over the shorter time interval, by a scaling factor inversely proportional to the square root of the ratio between intervals. So the probability P of an offset frequency fdev measured over an interval T is a function Pf_dev=P(f_offset)/SQRT(T/C). If the interval is quadrupled, the probability of measuring a given offset frequency over that longer interval is 1/2 the probability of measuring that frequency over the shorter interval.

However, the number of clock cycles employed in the frequency measurement increases linearly with the length of the interval. The measured frequency deviation (over the interval) represented by a single LSB of difference in clock cycles is inversely proportional to the number of clock cycles. So, longer time intervals permit more accurate measurement of a given frequency deviation. If an interval is quadrupled, the probability of a given offset frequency difference resulting in an LSB difference is four times the probability at the shorter interval.

In view of the above two paragraphs, it is clear that any amount of random frequency deviation can be measured so long as the interval is made long enough. The probability of measuring a given offset will decrease, but the ability of the measurement to detect that offset will increase faster than the decrease in probability of offset. In addition to this rather crude analysis, an additional factor may be the presence of low-frequency deviations in the Gaussian distribution. It is possible that the integration of the area under the Gaussian curve from zero offset to 1/fdev will increase to the point where a value of fdev=1/T will become likely.

The existence of a PLL to control the CPU clock and a separate crystal oscillator to control a real-time clock in every PC (and probably Macintosh) out there makes this method an extremely useful way to generate truly random bits.

FAQ
Q1: What about other pseudorandom variations caused by other interrupts, which may slow down the count in a predictable fashion?
A1: Uncorrelated variances add, so the presence of any truly random component in a binary number will result in a random LSB has long as the truly random component is at least one LSB in size.

Q2: How large would the count of CPU clock cycles be with a Pentium III running at 500 MHz and a 32.768 kHz interrupt rate from the real-time clock?
A2: 15,258.8

Q3: Obviously, you can't measure 0.8 clock cycles. You probably can't measure integer clock cycles either because of the count routine requiring multiple clock cycles. So what sort of resolution could you expect in that count?
A3: Probably a count by 3's, or 5086 possible count values. One for the increment, one for the branch back, one for unknown possibilities.

Q4: So what is the minimum frequency deviation (in Hertz) you could measure over one 30.5 microsecond interrupt interval from the real-time clock?
A4: 98 kHz.

Q5: What if the frequency offset is unlikely to be that large over any given interval, say only a 10% probability?
A5: Accumulate 100 potentially random bits over 100 real-time clock intervals. The chance of every single one of those intervals *not* producing a random bit is 0.90^100 = 0.0027%. Since at least one of the bits will almost certainly be random, simply XOR all the bits together and you'll have a random bit. Note that the specifications of the CK98 clock generator make it seem more likely than 10 percent that you'll see frequency deviations of 98 kHz.

If you actually use 200 intervals because of the need to set up before the next counting interval, the whole process would take six milliseconds per random bit produced.

14.51 Note (*computational efficiency of reduction modulo $b^t - c$*)

(i) Suppose that x has $2t$ base b digits. If $l \leq t/2$, then Algorithm 14.47 executes step 2 at most $s = 3$ times, requiring 2 multiplications by c. In general, if l is approximately $(s-2)t/(s-1)$, then Algorithm 14.47 executes step 2 about s times. Thus, Algorithm 14.47 requires about sl single-precision multiplications.

(ii) If c has few non-zero digits, then multiplication by c will be relatively inexpensive. If c is large but has few non-zero digits, the number of iterations of Algorithm 14.47 will be greater, but each iteration requires a very simple multiplication.

14.52 Note (*modifications*) Algorithm 14.47 can be modified if $m = b^t + c$ for some positive integer $c < b^t$: in step 2.2, replace $r \leftarrow r + r_i$ with $r \leftarrow r + (-1)^i r_i$.

14.53 Remark (*using moduli of a special form*) Selecting RSA moduli of the form $b^t \pm c$ for small values of c limits the choices of primes p and q. Care must also be exercised when selecting moduli of a special form, so that factoring is not made substantially easier; this is because numbers of this form are more susceptible to factoring by the special number field sieve (see §3.2.7). A similar statement can be made regarding the selection of primes of a special form for cryptographic schemes based on the discrete logarithm problem.

14.4 Greatest common divisor algorithms

Many situations in cryptography require the computation of the greatest common divisor (gcd) of two positive integers (see Definition 2.86). Algorithm 2.104 describes the classical Euclidean algorithm for this computation. For multiple-precision integers, Algorithm 2.104 requires a multiple-precision division at step 1.1 which is a relatively expensive operation. This section describes three methods for computing the gcd which are more efficient than the classical approach using multiple-precision numbers. The first is non-Euclidean and is referred to as the *binary gcd algorithm* (§14.4.1). Although it requires more steps than the classical algorithm, the binary gcd algorithm eliminates the computationally expensive division and replaces it with elementary shifts and additions. Lehmer's gcd algorithm (§14.4.2) is a variant of the classical algorithm more suited to multiple-precision computations. A binary version of the extended Euclidean algorithm is given in §14.4.3.

14.4.1 Binary gcd algorithm

14.54 Algorithm Binary gcd algorithm

INPUT: two positive integers x and y with $x \geq y$.
OUTPUT: $\gcd(x, y)$.

1. $g \leftarrow 1$.
2. While both x and y are even do the following: $x \leftarrow x/2$, $y \leftarrow y/2$, $g \leftarrow 2g$.
3. While $x \neq 0$ do the following:
   3.1 While x is even do: $x \leftarrow x/2$.
   3.2 While y is even do: $y \leftarrow y/2$.
   3.3 $t \leftarrow |x - y|/2$.
   3.4 If $x \geq y$ then $x \leftarrow t$; otherwise, $y \leftarrow t$.
4. Return$(g \cdot y)$.

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14.3.4 Reduction methods for moduli of special form

When the modulus has a special (customized) form, reduction techniques can be employed to allow more efficient computation. Suppose that the modulus m is a t-digit base b positive integer of the form $m = b^t - c$, where c is an l-digit base b positive integer (for some $l < t$). Algorithm 14.47 computes $x \bmod m$ for any positive integer x by using only shifts, additions, and single-precision multiplications of base b numbers.

14.47 Algorithm Reduction modulo $m = b^t - c$

INPUT: a base b, positive integer x, and a modulus $m = b^t - c$, where c is an l-digit base b integer for some $l < t$.
OUTPUT: $r = x \bmod m$.

1. $q_0 \leftarrow \lfloor x/b^t \rfloor$, $r_0 \leftarrow x - q_0 b^t$, $r \leftarrow r_0$, $i \leftarrow 0$.
2. While $q_i > 0$ do the following:
   2.1 $q_{i+1} \leftarrow \lfloor q_i c/b^t \rfloor$, $r_{i+1} \leftarrow q_i c - q_{i+1} b^t$.
   2.2 $i \leftarrow i+1$, $r \leftarrow r + r_i$.
3. While $r \geq m$ do: $r \leftarrow r - m$.
4. Return(r).

14.48 Example (*reduction modulo $b^t - c$*) Let $b = 4$, $m = 935 = (32213)_4$, and $x = 31085 = (13211231)_4$. Since $m = 4^5 - (1121)_4$, take $c = (1121)_4$. Here $t = 5$ and $l = 4$. Table 14.9 displays the quotients and remainders produced by Algorithm 14.47. At the beginning of step 3, $r = (102031)_4$. Since $r > m$, step 3 computes $r - m = (3212)_4$. □

i	$q_{i-1}c$	q_i	r_i	r
0	–	$(132)_4$	$(11231)_4$	$(11231)_4$
1	$(221232)_4$	$(2)_4$	$(21232)_4$	$(33123)_4$
2	$(2302)_4$	$(0)_4$	$(2302)_4$	$(102031)_4$

Table 14.9: *Reduction modulo $m = b^t - c$ (see Example 14.48).*

14.49 Fact (*termination*) For some integer $s \geq 0$, $q_s = 0$; hence, Algorithm 14.47 terminates.

Justification. $q_i c = q_{i+1} b^t + r_{i+1}$, $i \geq 0$. Since $c < b^t$, $q_i = (q_{i+1} b^t / c) + (r_{i+1}/c) > q_{i+1}$. Since the q_i's are non-negative integers which strictly decrease as i increases, there is some integer $s \geq 0$ such that $q_s = 0$.

14.50 Fact (*correctness*) Algorithm 14.47 terminates with the correct residue modulo m.

Justification. Suppose that s is the smallest index i for which $q_i = 0$ (i.e., $q_s = 0$). Now, $x = q_0 b^t + r_0$ and $q_i c = q_{i+1} b^t + r_{i+1}$, $0 \leq i \leq s-1$. Adding these equations gives $x + \left(\sum_{i=0}^{s-1} q_i\right) c = \left(\sum_{i=0}^{s-1} q_i\right) b^t + \sum_{i=0}^{s} r_i$. Since $b^t \equiv c \pmod m$, it follows that $x \equiv \sum_{i=0}^{s} r_i \pmod m$. Hence, repeated subtraction of m from $r = \sum_{i=0}^{s} r_i$ gives the correct residue.

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Application papers not suitable for publication

SN 10/092943 Mail Date 3/5/02

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  - ☐ Glossy pages
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  - ☐ Abstract
  - ☐ Claim(s)
- ☐ Drawing objection(s)
  - ☐ Missing lead lines, drawing(s)_____
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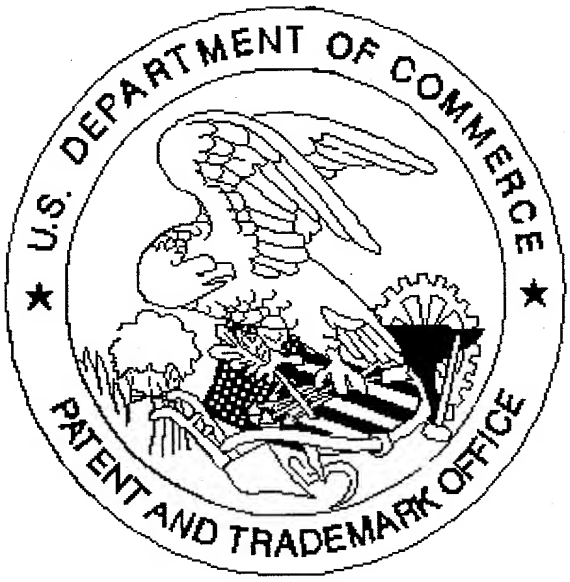

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